Worker Selection, Hiring and Vacancies*

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Abstract

This paper incorporates worker selection into a random matching model with multi-worker firms and studies its policy implications. Unlike the standard random matching model, the worker selection model is compatible with establishment-level behavior of the hires-to-vacancy ratio, which (i) steeply rises with the employment growth rate, (ii) falls with establishment size, and (iii) rises with worker turnover rate. I calibrate the worker selection model to match the salient features of the U.S. labor market and compare it with the standard matching model without worker selection model. I show that accounting for these patterns has both aggregate and firm-level implications for labor market policies. A hiring subsidy reduces aggregate unemployment substantially in the worker selection model, whereas the reduction in aggregate unemployment is very small in the standard model. Similarly, a firing tax reduces aggregate unemployment more in the worker selection model. At the firm level, labor market policy changes have a relatively bigger impact on fast growing and high worker turnover firms in the worker selection model. In contrast, the standard model implies that slowly growing and low worker turnover firms are affected relatively more by labor market policy changes. The existing empirical evidence supports the predictions of the worker selection model about the firm-level effects of labor market policies.

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1 Introduction

The impact of the Great Recession between 2007 and 2009 on the U.S. labor market was severe. The unemployment rate sharply increased at the onset of the recession, reaching as high as 10%, and still remains well above its pre-recession level. The gradual response of the unemployment rate has led to a debate about how to implement labor market policies to combat high U.S. unemployment rate. Currently, the most popular approach to labor market analysis in macroeconomic context is to use the framework of the Diamond-Mortensen-Pissarides (DMP) random matching model. While the standard DMP model is successful in many dimensions, it implies that the hires-to-vacancy ratio is constant across establishments. This implication is incompatible with the data. In the U.S., the hires-to-vacancy ratio at the establishment level (i) rises steeply with employment growth rate, (ii) falls with employer size, and (iii) rises with worker turnover rate. I extend the standard DMP model to resolve this discrepancy between the data and the theory and show that accounting for these patterns has important consequences for labor market policy analysis.

I incorporate worker selection into a random matching model where multi-worker firms hire among a pool of applicants. In response to idiosyncratic productivity shocks, firms post vacancies and are randomly matched to unemployed workers according to an aggregate matching function. Unlike the standard DMP model, not all of the matches are hired. Instead, firms go through a costly evaluation process of applicants before making a hiring decision. I model this worker selection process by allowing firms to partially observe the match quality of the applicants, set a minimum hiring standard, and hire only the applicants who satisfy this threshold. As firms select workers differently, the hires-to-vacancy ratio varies by establishment. The calibrated model accounts for all three patterns of the hires-to-vacancy ratio observed in the data.

Accounting for these patterns of the hires-to-vacancy ratio has both aggregate and firm-level policy implications. For example, when firms are subsidized for hiring new workers, the decline in unemployment rate is about seven times larger in the worker selection model. At the firm level, a hiring subsidy shifts the employment growth distribution to the right in both models, but most of this shift occurs in the right tail of the employment growth distribution in the worker selection model. Moreover, the worker selection model implies that firms that have initially high worker turnover rates experience a relatively larger decline in the worker turnover rates after a firing tax. In contrast, the standard model predicts that the worker turnover rate decreases more at firms with initially lower worker turnover rates. The firm-level implications of the worker selection model are consistent with the empirical evidence from the literature.
The cross sectional patterns of the hiring-vacancy ratio are documented in Davis, Faberman and Haltiwanger (2012), henceforth DFH, where the authors use establishment-level data from the Job Openings and Labor Turnover Survey (JOLTS). Building upon the cross sectional patterns of the hires-to-vacancy ratio, they argue that firms heavily rely on other instruments in addition to vacancies as they hire new workers. This paper adds worker selection as a new instrument for recruiting new workers. Such an extension is consistent with microeconomic evidence regarding firms’ hiring practices. Barron and Bishop (1985) report from the 1982 Employment Opportunities Pilot Project that company personnel spend on average 9.87 hours per hire to recruit, screen, and interview applicants. The standard deviation of the time spent per hire for evaluating applicants is 17.16 hours. These numbers imply that per-hire cost of recruitment is on average 4.3% of the quarterly wage of a newly hired worker and varies across firms.\(^1\) I interpret this variation in firms’ recruitment costs as evidence for worker selection introduced in this paper.

I motivate the idea of worker selection by introducing worker heterogeneity in the form of unobserved match-specific quality shocks. Firms first choose the number of vacancies and randomly matched with unemployed workers. Then, each vacancy-worker pair draws a match-specific quality shock which determines the productivity of a worker at the hiring firm. When increasing employment, firms now face a trade-off between the quantity and the quality of workers. Given a fixed number of vacancies, a firm would add fewer workers if it wanted to hire high quality workers. The decision for the hiring standard depends on how the quality and quantity margins interact.

Worker selection explains how the model generates the three patterns of the hires-to-vacancy ratio documented above. First, in a growing firm, the marginal cost of increasing the hiring standard is larger because a growing firm also posts more vacancies and contacts a larger group of applicants. Therefore, a growing firm fills vacancies faster by being less picky about new recruits and attains a higher hires-to-vacancy ratio. Second, as the firm size increases, the employment growth rate decreases and vacancies are filled at a slower rate. This mechanism generates a negative relationship between the firm size and the hires-to-vacancy ratio. Finally, a firm that has initially set a lower hiring standard lays off a larger fraction of the current recruits in the near future, which implies a positive relationship between the hires-to-vacancy ratio and the worker turnover rate. The calibrated model accounts for all of the three patterns of the hires-to-vacancy ratio at the same time.

The model in this paper links existing models of worker selection to those with multi-worker firms to account for the cross-sectional patterns in the hires-to-vacancy ratio. On the one hand, existing worker selection models are not suitable for studying the cross-sectional

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\(^1\)See Silva and Toledo (2009) and Hagedorn and Manovskii (2008).
properties of hires and vacancies, because they assume that firms either have a vacant position or are employed with only one worker. Mortensen and Pissarides (2001), Pries and Rogerson (2005), Villena-Roldan (2008) and Merkl and van Rens (2013) are examples of worker selection models of this kind. On the other hand, extensions to the standard DMP model assume workers are identical and, therefore, imply that firms indiscriminately hire all the workers they match. Then, for any firm, the job filling rate is determined by the aggregate matching function and is independent of individual firm’s characteristics. Consequently, there is no firm-level variation in the hires-to-vacancy ratio in these models. Examples of papers with multi-worker firms are relatively new in the literature and include Cahuc, Marque and Wasmer (2008), Elsby and Michaels (2010), Acemoglu and Hawkins (2010) and Fujita and Nakajima (2013).

When matching is not random, the hires-to-vacancy ratio may vary at firm level even without worker heterogeneity. For example, Kaas and Kircher (2011) build a directed search model, where firms attract workers by posting wages. Posting wages adds another competitive element into the labor market: firms that want to grow faster post higher wages in the market, attract more workers and fill vacancies at a higher rate. This generates a positive relationship between the hires-to-vacancy ratio and the employment growth rate. The worker selection and directed search models are complementary to each other in the way they model firm’s search activities. In the directed search model, firms’ search activities are at the extensive margin, i.e. firms can affect the total number of applicants by posting different wages. In contrast, firms in the worker selection model search at the intensive margin, i.e. firms can search for better workers within a pool of applicants.

Another notable difference between the worker selection and directed search models is that worker selection model can account for the cross-sectional behavior of the worker turnover rate. In the directed search model, faster growing firms hire more workers and hence experience a higher worker turnover rate. However, all of the growing firms separate from workers at a constant rate by assumption. Therefore, the relationship of the hires-to-vacancy ratio to the worker turnover rate is only a restatement of its relationship to the employment growth rate. On the contrary, the separation rate is not constant in the worker selection model. The worker selection model asserts that firms that fill vacancies at a faster rate experience a higher separation rate, because they hire proportionally more low-quality workers. Since the job filling rate is positively related to employment growth rate, the worker selection model predicts that the separation rate steadily rises in the cross section moving

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2 Villena-Roldan (2008) differs from the others by allowing firms to meet multiple workers. However, firms are still restricted to hire at most one worker.

3 To be precise, the authors consider only the equilibrium in which wage contracts specify a constant separation rate for all of the workers within a firm.
from stable to growing firms. According to the findings of DFH, separation rate increases from 1% to 5% as employment growth rate rises from 0% to 25% in the cross section, which supports the prediction of the worker selection model.\footnote{Matching the establishment-level behavior of worker turnover is novel to the worker selection model introduced in this paper. This feature of the model distinguishes it not only from the directed search model, but also from other models with establishment-level variation in the hires-to-vacancy ratio that assume a constant separation rate in expanding firms. See DFH for a discussion of other models that can potentially account for the behavior of the hires-to-vacancy ratio.}

In the policy analysis section, I use the calibrated model to examine the aggregate and firm-level effects of a hiring subsidy and a firing tax. A hiring subsidy is a per-hire payment made to firms that hire new workers. A firing tax is a payment collected from firms for each worker they fire. To highlight the impact of worker selection, I compare the results obtained from the worker selection model to those obtained from the standard DMP model with homogeneous workers calibrated to match the same targeted moments.

A hiring subsidy and a firing tax affect the aggregate labor market outcomes differently with and without worker selection. For example, if employers are paid half of the average wage of a newly hired worker, the unemployment rate in the worker selection model falls by half of a percentage point. In contrast, the standard DMP model is not very optimistic about the effects of the hiring subsidy on the unemployment rate. The decline in unemployment rate in response to an equivalent subsidy in the standard DMP model is one-seventh the size of the decline in the worker selection model. While a firing tax increases the aggregate unemployment rate only slightly in the standard DMP model, the effect in the worker selection model is about ten times larger.

The worker selection and the standard DMP models also produce different results about the aggregate hires-to-vacancy ratio. In the standard DMP model, changes in the aggregate hires-to-vacancy ratio in response to policy changes are unambiguous, because the direction of the change is solely determined by the aggregate matching function. However, in the worker selection model, firms also respond to the policy changes by adjusting their hiring standards, which changes the aggregate hires-to-vacancy ratio in the opposite direction. The net effect on the aggregate hires-to-vacancy ratio depends on which of the quality and quantity margins dominates the effect of the other. For example, in response to a hiring subsidy the aggregate hires-to-vacancy ratio unambiguously falls in the standard DMP model due to increased number of total vacancies. In the worker selection model, a hiring subsidy also makes firms less picky about potential hires, which tends to increase the aggregate hires-to-vacancy ratio. When the hiring subsidy is small, the quality margin dominates the quantity margin and the aggregate hires-to-vacancy ratio rises. As the size of the hiring subsidy increases, the aggregate hires-to-vacancy ratio starts to decline as the quantity margin dominates the
quality margin. Similarly, a firing tax increases the aggregate hires-to-vacancy ratio in the
standard DMP model due to decreased number of total vacancies. In the worker selection
model, a firing tax also induces firms to be more picky about the workers as they have to
now pay an additional cost for hiring a low-productive worker. I find that the aggregate
hires-to-vacancy ratio declines with a firing tax.

Both models produce similar results regarding the effect of these policies on aggregate
output net of labor adjustment costs. A firing tax always reduces net output, because too
few firms produce in equilibrium due to increased labor adjustment costs. On the other
hand, a hiring subsidy can increase net output. However, if the hiring subsidy becomes
too large, firms start replacing existing workers with new workers and experience very large
worker turnover rates. Further increases in the hiring subsidy eventually reduces net output,
because firms incur large adjustment costs, but contribute only a little to the aggregate
output.

Earlier papers that used matching models to analyze labor market policies are silent
about the effects of labor market policies on firms with different characteristics. Using a
multi-worker setting in this paper, I provide new insights about the firm-level effects of labor
market policies. Contrary to the predictions of the standard DMP model, worker selection
makes fast growing and large worker turnover firms respond relatively more to changes in
labor market policies.

A hiring subsidy creates incentives for hiring new workers and shifts the employment
growth distribution to the right in both models. However, a hiring subsidy in the standard
DMP model has a relatively bigger impact on employment at slowly growing firms, while
the opposite is true in the worker selection model. Therefore, a hiring subsidy shifts the
employment growth distribution out in the right tail in the worker selection model. The im-
plication of the worker selection model is consistent with the findings of Perloff and Wachter
(1979). They argue that The New Jobs Tax Credit of the stimulus package in 1977 shifted
the employment growth distribution of the firms who knew about the subsidy program to
the right relative to those who did not know about the program. They further argue that
most of this shift occurred in the right tail of the distribution.

In response to a firing tax, the worker selection model predicts that worker turnover
rates falls relatively more at firms with initially high worker turnover rates. Conversely, the
standard DMP model predicts that the effect of a firing tax on the worker turnover rate
is larger at firms with initially lower worker turnover rates. In a cross-country comparison,
Haltiwanger, Scarpetta and Schweiger (2010) show that firms in the industries and size

\footnote{Some low productive firms even grow at a slower rate after a hiring subsidy, because hiring is more costly
due to increased number of aggregate vacancies.}
classes that require more often employment changes are affected relatively more from hiring and firing restrictions. This evidence is consistent with the predictions of the worker selection model about the effects of a firing tax.

The paper is organized as follows. Section 2 describes the worker selection model and Section 3 characterizes the equilibrium. I calibrate the model in Section 4 and discuss my findings in Section 5. Section 6 presents the results from the counterfactual policy experiments with a hiring subsidy and a firing tax. The last section concludes.

2 The Worker Selection Model

2.1 Overview

The economy is populated by risk-neutral workers, the measure of which is normalized to 1, and a large number of risk-neutral entrepreneurs. Time is discrete and the discount factor for both the workers and the entrepreneurs is $\beta$. Each entrepreneur runs a firm which produces a single good. Hereafter, I refer to firms and entrepreneurs interchangeably. There are no aggregate shocks and the focus is on the steady state equilibrium.

In any period, a worker is either employed or unemployed. An employed worker receives a wage income, but there is no on-the-job-search. An unemployed worker searches for a job; if he cannot find a job, he is engaged in home production and receives $b$. Workers consume all of their income in the current period.

In any period, a firm can be either active or inactive. An active firm employs a measure of workers denoted by $n$. Firm productivity has an idiosyncratic component, $\varepsilon$. It evolves according to a Markov process, $F(\varepsilon' | \varepsilon)$, where I adopt prime notation to denote variables in the next period. The productivity process is common to all of the firms. An inactive firm can become active at the beginning of each period by paying a fixed entry cost, $c_e$. Upon entry, it draws its initial idiosyncratic productivity from the unconditional distribution of the same Markov process, $F_0(\varepsilon)$. Active firms become inactive with exogenous probability $\delta$. Productivity shocks are large enough to ensure that none of the firms optimally chooses to become inactive at any point in time.

2.2 Recruiting New Workers

Recruiting new workers consists of three stages: vacancy posting, worker selection and wage bargaining. The first and the last stages are common to the standard DMP matching model. The innovation of this paper is the introduction of the interim stage where firms selectively hire among a pool of applicants.
In the first stage, firms post vacancies, \( v \), to attract unemployed workers and pay \( c_v \) per vacancy. There are matching frictions in the labor market. Total number of matches in the economy is determined via an aggregate matching function, which has a CES form:

\[
M(U, V) = (U^{-\zeta} + V^{-\zeta})^{-\frac{1}{\zeta}}
\]  

(1)

\( U \) and \( V \) are total number of unemployed workers and vacancies, respectively. \( \zeta > 0 \) governs the degree of elasticity of substitution. Let \( \theta = V/U \) be the market tightness. Then, a firm that posts \( v \) number of vacancies meets \( q(\theta)v \) workers, where \( q(\theta) \) is the probability that a vacancy meets a worker. \( q(\theta) \) is derived from the matching function as follows:

\[
q(\theta) = \frac{M(U, V)}{V} = (1 + \theta^\zeta)^{-\frac{1}{\zeta}}
\]  

(2)

Similarly, \( M(U, V)/U = \theta q(\theta) \) is the probability a worker meets a vacancy.\(^6\) In the sequel, I drop \( \theta \) and simply write \( q \) for notational purposes.

In the second stage, each worker matched with a firm draws an unobserved match-specific quality shock, \( x_i \), from a uniform distribution between 0 and 1. The match-specific quality shock determines whether the worker will be productive or unproductive at the hiring firm conditional on being hired. Specifically, a worker with a match-specific quality \( x_i \) becomes productive at the hiring firm with probability \( x_i^{\gamma-1} \), where \( \gamma > 1 \). Otherwise, the worker becomes unproductive. Both the firm and the worker learn the true productivity of the worker only after one period of employment. If a worker turns out to be unproductive, he leaves the firm. Although the match-specific quality shock is unobserved at the time of hiring, the firm can engage in a costly process where it evaluates the applicants and infer about their true match-specific quality. I model this process by allowing firms to choose a hiring standard, \( p \in [0, 1] \), and hire only the worker that satisfy this minimum threshold. Firms observe the match-specific quality of an applicant up to \( p \). If an applicant’s match-specific quality is greater than \( p \), it is still unobserved, but known to be greater than \( p \).

Total selection costs have the following quadratic form:

\[
C_s(p, qv) = \frac{c_s}{2} \exp\left(\frac{c_p}{2} p^2\right) (qv)^2
\]  

(3)

This functional form assumes that worker selection technology exhibits decreasing returns to scale in the number of applicants captured by the quadratic term: given \( p \), the marginal cost of selection is increasing in the number of applicants. It also assumes that this marginal cost is increasing in the hiring standard set by the firm. The interpretation is that worker

\(^6\)Note that \( \zeta > 0 \) guarantees that both of the meeting probabilities lie in the interval \([0,1]\).
selection is time-consuming and as the hiring standard goes up the interviewer spends more

time to distinguish high quality workers from low quality ones. The exponential form in

\( p \) satisfies three conditions. First, it is greater than zero at \( p = 0 \), which prevents firms
to choose \( p \) close to zero, post enormous amount of vacancies and converge to their long-

run employment level in a short period of time. Second, the derivative of this function with

respective \( p \) is zero at \( p = 0 \). This property of the exponential function guarantees an interior

solution for \( p \). Finally, it is log-convex, which guarantees that the dynamic programming

problem of a hiring firm is concave.

Due to matching frictions in the labor market, a firm’s current match with its workers

generates bilateral monopoly rents. In the third stage, firms bargain over the wage with

their existing workers and the workers in their applicant pool to split these rents. I describe

wage bargaining formally below.

I refer to the second stage above as worker selection, because the wage bargaining process

implies that a firm does not hire all the workers it matches. To see that, consider an applicant

with \( x_i = 0 \). His contribution to output is zero in this period and he leaves the firm at the end

of the period. However, the firm has to compensate for his outside option, i.e. value of finding

a job with a higher match quality. The total value of surplus from this match is negative

and both parties mutually agree not to form an employment relationship. Furthermore, the

value of a worker to the firm increases with \( x_i \). Hence, there exists a reservation match-
specific quality below which workers have negative value to the firm.\(^7\) The firm identifies

those workers during the worker selection process.

\[ \text{2.3 Firms’ Problem} \]

Since firms are subject to idiosyncratic productivity shocks, large firms that receive an

adverse productivity shock may find it optimal to reduce employment. However, such a firm

would never find it optimal to hire from the unemployment pool, because an existing worker

is more productive than any potential new worker and adjustment is costly. Moreover, the

problem of a hiring firm includes an additional control variable, \( p \). Therefore, I write down

the dynamic problem of a hiring and firing firm separately. I allow for a corner solution for

the firing firm when it neither hires nor fires any worker.

Let \( \Pi^h(n, \varepsilon) \) and \( \Pi^f(n, \varepsilon) \) denote the value of a hiring and firing firm, respectively. Let

also \( \Pi(n, \varepsilon) = \max(\Pi^h(n, \varepsilon), \Pi^f(n, \varepsilon)) \). Given the timing of events in a period, total number

of hires at a firm posting \( v \) vacancies are equal to \( qv(1 - p) \), but only \( qv(1 - p^\gamma)/\gamma \) will be

productive and retained by the firm next period. Hence, employment at a hiring firm evolves

\(^7\)Some of the newly hired workers will have negative value to the firm. However, due to costly selection,

the firm does not attempt to find those workers in the applicant pool.
over time according to the following equation:

\[ n' = (1 - \lambda)n + qu(1 - p^\gamma)/\gamma \quad (4) \]

I assume that incumbent workers lose their jobs at the beginning of the period with probability \( \lambda \) and can search for a new job in the current period.

An active firm has access to a Cobb-Douglas production function which depends on the number of productive workers employed in the current period: \( A \varepsilon n^{\alpha} \). Note that the production function accounts for the fact that new recruits are employed in the current period.

Let \( w^n(n', \varepsilon) \) and \( w^p(n', \varepsilon, p) \) denote wages paid to existing workers and new recruits, respectively. I conjecture and verify later that both wages depend on the total number of productive workers and firm productivity. In addition, the hiring standard affects the wage payment to the new workers as it determines the expected productivity of a randomly selected new hire. This specification is later verified in the wage bargaining section.

The following summarizes the dynamic programing problem of a hiring firm:

\[
\Pi^h(n, \varepsilon) = \max_{n', p \in [0, 1], v \geq 0} \left[ -c_v v - \frac{c_s}{2} \exp \left( \frac{cp^2}{2} \right) (qv)^2 + A \varepsilon n^{\alpha} - qu(1 - p)w^p(n', \varepsilon, p) - (1 - \lambda)n w^n(n', \varepsilon) + \beta (1 - \delta) E_{\varepsilon'|\varepsilon}[\Pi(n', \varepsilon')] \right] (5)
\]

subject to (4).

Let \( f \) denote total firings. Then, employment at a firing firm evolves according to:

\[ n' = (1 - \lambda)n - f \quad (6) \]

The dynamic programing problem of a firing firm is as follows:

\[
\Pi^f(n, \varepsilon) = \max_{n', f \geq 0} \left[ A \varepsilon n^{\alpha} - n' w^n(n', \varepsilon) + \beta (1 - \delta) E_{\varepsilon'|\varepsilon}[\Pi(n', \varepsilon')] \right] (7)
\]

subject to (6).

### 2.4 Worker’s Value Functions

Let \( \tilde{V}^u \) and \( V^u \) denote the value of unemployment at the beginning of the period and after the labor market closes, respectively. I describe how they are related to each other further below. The value function of an existing worker employed at a firm with \( n \) workers and
productivity $\varepsilon$ is:

$$V^n(n, \varepsilon) = w^n(n', \varepsilon) + \beta((1 - \delta)((1 - \lambda)E_{\varepsilon'|\varepsilon}[V^n(n', \varepsilon')] + \lambda\tilde{V}^u) + \delta\tilde{V}^u)$$

where $n'$ is the firm’s optimal decision for employment and depends on current size, $n$, and productivity, $\varepsilon$, of the firm. The worker takes $n'$ as given. The interpretation is standard: an existing worker receives $w^n(n', \varepsilon)$ this period. With probability $(1 - \delta)(1 - \lambda)$, he is employed at the same firm and enjoys the expected value of employment. Otherwise, he receives $\tilde{V}^u$. Note that the expected value of employment is over the productivity shocks and accounts for the change in firm’s employment.

Let $g(p) = \frac{1 - p^\gamma}{\gamma(1 - p)}$ be the probability that a randomly selected new hire is productive. Then, the value function of a newly hired worker is:

$$V^p(n, \varepsilon) = w^p(n', \varepsilon) + \beta(g(p)((1 - \delta)((1 - \lambda)E_{\varepsilon'|\varepsilon}[V^n(n', \varepsilon')] + \lambda V^u) + \delta V^u)$$

Note that the continuation value of a newly hired worker depends on the hiring standard set by the firm this period, $p(n, \varepsilon)$, and is taken as given by the worker. The functional equation above is otherwise same with (8). Finally, $V^u$ and $\tilde{V}^u$ are related according to:

$$V^u = b + \beta\tilde{V}^u$$

$$\tilde{V}^u = \theta q \int_{\varepsilon, \mathcal{N}} \frac{v(n, \varepsilon)((1 - p(n, \varepsilon))V^p(n, \varepsilon) + p(n, \varepsilon)V^u)}{\int_{\varepsilon, \mathcal{N}} v(n, \varepsilon)d\Gamma(n, \varepsilon)}d\Gamma(n, \varepsilon) + (1 - \theta q)V^u$$

where $\Gamma$ is a probability measure of firms over $(n, \varepsilon)$ and $\mathcal{N}$ and $\mathcal{E}$ are sets of all possible realizations of $n$ and $\varepsilon$, respectively. At the beginning of the period, an unemployed worker matches with a vacancy with probability $\theta q$. Conditional on a match, he receives the expected value of the outcome of the selection process: with probability $(1 - p(n, \varepsilon))$ he is employed and enjoys the value of being employed at a firm with $n$ workers and productivity $\varepsilon$. Otherwise, he is unemployed and receives $V^u$. The probability that he matches with a firm of size $n$ and productivity $\varepsilon$ is weighted by the firm’s share of vacancies in total vacancies. Finally, with probability $(1 - \theta q)$, he does not find a match and receives $V^u$.

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8In fact, (8) can be thought as the limiting case of (9) as $p \to 1$. 

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2.5 Wage Bargaining

To determine wages, I adopt the bargaining solution in Stole and Zwiebel (1996). They describe a dynamic game where the firm negotiates the wage payment in pairwise bargaining sessions with its employees in an arbitrary order. If an agreement is reached between the worker and the firm during a bargaining session, the firm continues bargaining with the next worker. Otherwise, the worker leaves the firm and the bargaining process resumes with all the remaining workers. Each bargaining session is the limiting case of the offer-counteroffer game between the firm and the worker described in Binmore et al. (1986). In this offer-counteroffer bargaining game, each time the worker rejects an offer, there is an exogenous probability, \((1 - \phi)d\), that the negotiations break down. Similarly, each time the firm rejects an offer, the negotiations break down with probability \(\phi d\). As \(d \to 0\), they split the joint surplus net of outside options such that the worker receives \(\phi\) fraction of it. If there is only one worker, the solution is the Nash bargaining solution with \(\phi\) being the workers’ bargaining power. For the firm, the surplus is continuing the bargaining process with one less worker. When labor is continuous, the solution to the wage function implies a split of the marginal surplus and outside option of the worker according to bargaining powers.

The bargaining game in Stole and Zwiebel (1996) assumes that workers are the same with respect to their productivities. In the worker selection model, existing and new workers differ in size and productivity and are potentially paid different wages. The firm negotiates with \((1 - \lambda)n\) existing workers and \(qv(1 - p)\) potential workers. The productivity of existing workers is \(1\) and the productivity of selected applicants is \(g(p)\). Let me define total surplus to the firm at the bargaining stage as \(D(\tilde{n}, r, p, \varepsilon)\), where \(\tilde{n} = (1 - \lambda)n\) and \(r = qv(1 - p)\).

At the bargaining stage, vacancy posting and worker selection costs are sunk. Hence, from the firms problem, one obtains total surplus as follows:

\[
D(\tilde{n}, r, p, \varepsilon) = A\varepsilon(\tilde{n} + g(p)r)^\alpha - w^n(\tilde{n} + g(p)r, \varepsilon)\tilde{n} - w^p(\tilde{n} + g(p)r, \varepsilon)r + \beta(1 - \delta)E_{\varepsilon|\varepsilon}[\Pi(\tilde{n} + g(p)r, \varepsilon)]
\]  

(12)

Note that \(n'\) is equal to \(\tilde{n} + g(p)r\). The marginal surplus to the firm from an existing worker is the partial derivative of the total surplus with respect to \(\tilde{n}\), \(D_\tilde{n}(\tilde{n}, r, p, \varepsilon)\). The marginal surplus to the firm from a potential worker is similarly given by \(D_r(\tilde{n}, r, p, \varepsilon)\). Then, the solution to the bargaining problem satisfies the following conditions:

\[
\phi D_\tilde{n}(\tilde{n}, r, p, \varepsilon) = (1 - \phi)(V^n(n', \varepsilon) - V^u)
\]  

(13)

\[
\phi D_r(\tilde{n}, r, p, \varepsilon) = (1 - \phi)(V^p(n', \varepsilon) - V^u)
\]  

(14)
Using these two conditions along with the firm’s problem and workers’ value functions, I obtain the wage functions for each group as follows:

\[ w^n(n',\varepsilon) = \frac{\alpha \phi}{1 - \phi + \alpha \phi} A \varepsilon n'^{\alpha - 1} + (1 - \phi)\Omega \quad (15) \]

\[ w^p(n',\varepsilon,p) = g(p) \frac{\alpha \phi}{1 - \phi + \alpha \phi} A \varepsilon n'^{\alpha - 1} + (1 - \phi)\Omega \quad (16) \]

where \( \Omega \) is:

\[ \Omega = (1 - \beta)V^u - (\lambda + \delta - \lambda \delta)((1 - \beta)V^u - b) \quad (17) \]

The derivations are available in Appendix A. The wage functions are similar to ones obtained in other papers featuring random matching with multi-worker firms, e.g., Acemoglu and Hawkins(2010), Elsby and Michaels(2012) and Cahuc, Marque and Wasmer (2008). The solution for the wage equation in Stole and Zwiebel (1996) implies sharing of the worker’s outside option and the weighted average of infra-marginal products of labor. The solution above preserves this property except that it now includes additional terms to the worker’s outside option due to timing assumption. The wages at a non-hiring firm (firing or no-action) is the same as \( w^n(n',\varepsilon) \). Further, wages at a firing firm are such that \( V^n(n',\varepsilon) = V^u \). This is implied by equation (13). Finally, as I conjectured, both wage functions depend only on the total number of productive workers and not separately on the number of productive and unproductive workers. This result is not surprising given that both groups enter the production function linearly.

### 2.6 Recursive Stationary Equilibrium

Two more conditions are needed to define the recursive stationary equilibrium. First, \( \Gamma(n,\varepsilon) \) must be consistent with firms’ optimal decision for employment at the steady state. Hence, it satisfies:

\[ \Gamma(N, E) = \int_{N, E} \left[ \int_{\mathcal{N}, \mathcal{E}} F(\varepsilon'|\varepsilon)\mathcal{I}(n' = n'(n,\varepsilon))d\Gamma(n,\varepsilon) \right] dn'd\varepsilon' \quad (18) \]

where \( N \subset \mathcal{N}, E \subset \mathcal{E} \) and \( \mathcal{I} \) is an indicator function which is 1 if the condition is satisfied and 0 otherwise.

Second, the recursive stationary equilibrium satisfies a free entry condition given by:

\[ E_\varepsilon(\Pi(0,\varepsilon)) = c_\varepsilon \quad (19) \]

A formal definition of recursive stationary equilibrium is available in Appendix B. Two
equilibrium outcomes, the measure of firms and the total number of unemployed workers seeking for jobs, are not specified in the definition of the recursive competitive equilibrium and can be calculated from other endogenous variables as follows. Let $\mu$ denote the mass of firms in equilibrium. Then, total vacancies and total unemployed workers are:

$$V = \mu \int_{\mathcal{N}, \mathcal{E}} v(n, \varepsilon) d\Gamma(n, \varepsilon)$$

$$U = 1 - (1 - \lambda - \delta) \mu \int_{\mathcal{N}, \mathcal{E}} nd\Gamma(n, \varepsilon) + \mu \int_{\mathcal{N}, \mathcal{E}} f(n, \varepsilon) d\Gamma(n, \varepsilon)$$

Recall market tightness is $\theta = V/U$. Using the equilibrium value of $\theta$ and the calculated decision rule for firings, one can obtain the equilibrium value of $\mu$. Plugging $\mu$ in the second equation above, equilibrium unemployment is determined.

### 3 Characterization of Equilibrium

Heterogeneity in firms’ recruiting practices is the main focus of this paper. Therefore, I analyze the problem of a hiring firm in this section. The problem of a firing firm is rather standard. Inserting the wage functions in the hiring firm’s optimization problem, the dynamic programming problem becomes:

$$\Pi^h(n, \varepsilon) = \max_{n', p \in [0, 1], v \geq 0} -c_v v - \frac{c_s}{2} \exp\left(\frac{c_p p^2}{2}\right) (qv)^2 + \frac{1 - \phi}{1 - \phi + \alpha \phi} A \varepsilon n'^\alpha$$

$$- (1 - \phi) \Omega ((1 - \lambda)n + (1 - p)qv)$$

$$+ \beta (1 - \delta) E'\varepsilon_{|\varepsilon, [\Pi(n', \varepsilon')]}$$

subject to (4).

#### 3.1 Optimal Decision for the Hiring Standard

In (20), when $\gamma = 1$, any worker from the unemployment pool is productive. Hence, firms optimally choose to hire every worker they match, i.e. $p(n, \varepsilon) = 0$ for all $(n, \varepsilon)$. In this case, the model reduces to the standard DMP model with multi-worker firms. In general, replacing $qv$ from (4) into the firm’s problem in (20) and taking the derivative with respect
to $p$ implies:

$$
(1 + (\gamma - 1)p^\gamma - \gamma p^{\gamma - 1})((1 - \phi) \Omega
= \gamma p^{\gamma - 1} c v / q + \frac{c_s}{2} \exp \left(\frac{c_p}{2} p^2\right) \gamma \Delta \left(\frac{1}{1 - p^\gamma}\right)^2 \left(c_p p(1 - p^\gamma) + 2\gamma p^{\gamma - 1}\right)
$$

where $\Delta = n' - (1 - \lambda)n$ is the net change in employment.\(^9\) In other words, the decision for the optimal hiring standard is a solution to static problem given $\Delta$. The LHS in (21) is strictly decreasing in $p$ and is equal to 0 when $p = 1$. This term is the marginal benefit from increasing the hiring standard: as a firm increases the hiring standard, it avoids paying wages to the workers who are more likely to be unproductive in the next period. However, this gain diminishes with $p$ as the firm has to post more vacancies to satisfy a given level of $\Delta$. The RHS, on the other hand, is strictly increasing in $p$ and equal to 0 when $p = 0$. This term is the marginal cost of increasing the hiring standard: as a hiring firm increases the hiring standard, the marginal cost of selection increases not only because the selection costs are larger when $p$ is larger, but also because the firm has to post more vacancies to satisfy a given level of $\Delta$. I plot these curves for $\Delta = 1$ in Figure 1 using the calibrated parameter values.\(^10\) As implied by LHS and RHS being monotone, the solution to $p$ is interior and unique.

Now, consider an increase in $\Delta$, i.e. the firm grows faster. This shifts the marginal cost curve up and leaves the marginal benefit unchanged. Such a change is depicted in Figure 1. Given the initial size of the firm, the optimal choice for $p$ falls with employment growth. Hence, if the firm grows faster, it fills vacancies faster and attain a high hires-to-vacancy ratio. However, this result is conditional on the initial and next period’s employment. The cross sectional patterns of the job filling rate depends on the optimal decision for employment in the next period, which I analyze in the next section.

### 3.2 Optimal Decision for Employment

The previous section characterizes the optimal decision for $p$. When $\Delta$ is given, the optimal decision for $p$ is independent of the production in the current period and the continuation value of the firm. This property of the optimal hiring standard allows me to characterize the adjustment cost function in terms of $\Delta$ given that $p$ is optimally chosen. Let $C(\Delta)$ be the

---

\(^9\)The common term $\frac{-\Delta}{(1-p^\gamma)^2}$ is factored out.

\(^10\)Since the factored out term includes $\Delta$, these curves represent marginal benefit and cost from increasing the hiring standard per net employment change.
Figure 1: Optimal Choice for $p$

The total cost to the firm from changing the employment from $n$ to $n'$. It is the value function of the following minimization problem:

$$C(\Delta) = \min_{p \in [0,1]} \left\{ \frac{c_v}{q} \frac{\gamma \Delta}{1 - p^\gamma} + \frac{c_s}{2} \exp \left( \frac{c_p}{2} p^2 \right) \left( \frac{\gamma \Delta}{1 - p^\gamma} \right)^2 + (1 - \phi) \Omega \gamma \Delta \frac{1 - p}{1 - p^\gamma} \right\}$$

I obtained the following results regarding the problem in (22). A detailed analysis is available in the Appendix C.

1. Let $p(\Delta)$ be the policy function in (22). Then, $p'(\Delta) < 0$, as discussed in the previous section.

2. $C''(\Delta) > 0$, i.e. the adjustment cost function is increasing.

3. $C'''(\Delta) > 0$, i.e. the adjustment cost function is strictly convex. This result uses the fact that $\exp \left( \frac{c_p}{2} p^2 \right)$ is log-convex in $p$.

The convexity of the adjustment cost function implies that the dynamic programming problem of a hiring firm is concave and the first order condition with respect to $n'$ is necessary and sufficient for optimal employment in the next period.
The convexity of the adjustment cost function also determines the relationship of hires-to-vacancy ratio to employment growth, firm size and worker turnover, conditional on productivity. It implies that an entrant firm gradually converges to its long-run size. Therefore, small firms post more vacancies, grows faster, fills vacancies faster and attain a higher hires-to-vacancy ratio. This establishes the relationship of the hires-to-vacancy ratio to employment growth and firm size. Small firms also experience larger worker turnover rates because they set lower hiring standards and separate from the newly hired workers in the next period with a greater likelihood. This generates a positive relationship between the hires-to-vacancy ratio and the worker turnover rate. All of these three results about the behavior of the hires-to-vacancy ratio are conditional on productivity. Using the calibrated model, I show that these results also hold in the cross section.

4 Calibration

I calibrate the model to match the salient features of JOLTS documented in DFH. Unless otherwise stated, all the targeted moments are taken from their work. The parameter estimates are presented in Table 1.

I choose a period to be equal to one week and I set the discount factor to match the quarterly interest rate of 1.12%. As in Acemoglu and Hawkins (2010) and Fujita and Nakajima (2013), I use 0.67 for the curvature of the Cobb-Douglas production function. This value is commonly used in the real business cycle literature and is a lower bound when decreasing returns are due to factors other than labor that are fixed. For worker’s bargaining power, Shimer (2005) and Hagedorn and Manovskii (2008) use 0.72 and 0.052, respectively. Shimer (2005) justifies his choice for the bargaining parameter by relying on the Hosios condition, which does not apply in this paper. Hagedorn and Manovskii (2008) target the elasticity of wages with respect to productivity to calibrate worker’s bargaining power. I use an intermediate value and assume equal bargaining power between the firm and the workers.

The idiosyncratic productivity process approximates an AR(1) process:

\[ \log(\varepsilon_{t+1}) = \rho \log(\varepsilon_t) + \eta_t \]  

(23)

with \( \eta_t \sim N(0, \sigma^2) \). For the persistence parameter, \( \rho \), I use the estimate in Abraham and White (2006). They find the persistence of the idiosyncratic shocks to be 0.59 on an annual basis.\(^{11}\) To represent this process on a weekly basis, I impose that firms get a productivity

\(^{11}\)This value is the estimate without the firm fixed effects. To be consistent with the specification in (23), I use this value throughout the analysis. After controlling for the firm fixed effects, Abraham and White (2006) estimate the persistence parameter as 0.40. Changing the value of \( \rho \) this value does not affect the
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
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<td>$\phi$</td>
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<td>Persistence of idiosyncratic shocks</td>
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<td>Dispersion of idiosyncratic shocks</td>
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<td>$\delta$</td>
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<td>$\zeta$</td>
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</tr>
<tr>
<td>$c_p$</td>
<td>Selection cost- quality margin</td>
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<td>$A$</td>
<td>Aggregate productivity</td>
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<tr>
<td>$c_e$</td>
<td>Fixed entry cost</td>
<td>3154.6083</td>
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</table>

Table 1: Calibrated Parameters of the Worker Selection Model (Weekly Model)

shock with probability 1/52 in a given week. I choose the variance of the shocks to match a hires rate of 3.4%.

There are three sources of worker-firm separation in the model. First, firms fire productive workers in response to a negative productivity shock, so separations due to firings are driven by the productivity process. Since separations are equal to hires in a stationary equilibrium, I account for this type of separation by setting $\sigma$ to match the hires rate. Second, some of the newly hired workers leave the firm next period if they turn out to be unproductive. The probability that a worker with the average match-quality will be productive next period depends on $\gamma$. All else equal, when $\gamma$ becomes larger, a larger fraction of the newly hired workers leave the firm next period. Hence, a larger value of $\gamma$ implies a larger difference between the worker turnover rate, which is the sum of hiring and separation rates, and the job turnover rate, which is the sum of net job creation and destruction rates. In JOLTS, the monthly job turnover rate is 3.0%, less than half of the worker turnover rate. Since the hires rate is already targeted, I choose $\gamma$ to match the monthly job turnover rate. Finally, properties of the hires-to-vacancy ratio in the cross section.
separations occur exogenously with probability $\lambda$ or due to firm exit with probability $\delta$. In JOLTS, separations due to reasons other than quits and lay-offs is 0.24%. I set $\lambda$ to this value. Consistent with the evidence from Davis, Haltiwanger, and Schuh (1996), I choose $\delta$ so that one-sixth of job destruction is due to firm exit. JOLTS excludes exiting firms. Accordingly, I set $\delta = 0.015/5 = 0.003$.

The value of $b$ relative to the average worker productivity, $Y/N$, plays an important role in the context of the volatility puzzle. A higher value of the ratio of $b$ to $(Y/N)$ tends to amplify the effects of productivity shocks in the standard DMP model. The values used in the literature lies between 0.4 and 0.955. Following Mortensen and Nagypal (2007), I set the ratio of $b$ to $Y/N$ to 0.72. Furthermore, I normalize the equilibrium value of $\Omega$ to 1 and choose $A$ to satisfy this equilibrium value.

Note from equation (21) that, as the number of vacancies posted approaches zero, the optimal hiring standard approaches a value that is strictly less than 1. Given $\gamma$ and $V^u$, the magnitude of $c_v$ determines this upper bound for the optimal hiring standard. In the lowest worker turnover quintile, the daily job filling probability is equal to 0.011. A similar value is observed around the zero employment growth rate. In weekly terms, this is equal to 0.0745. Accordingly, I choose $c_v$ so that the job filling probability is equal to 0.0745 in the model when total vacancies are equal to zero.

The daily job-filling rate in the data is 0.05. Hence, the probability of filling a vacancy in a week is 0.3017. The model counterpart of this value is $q(1 - \bar{p})$, where $\bar{p}$ is the average hiring standard set by the firms. Shimer (2005) estimates that the average job finding probability of a worker in a month is 0.45. In weekly terms, this is equal to 0.1388. In the model, this is given by $\theta q(1 - \bar{p})$. Dividing the latter by the former, I obtain $\theta = 0.4601$. To determine $q$, I use the fact from Roldan-Vilena (2008) that firms interview, on average, five applicants before filling an open position. This value implies that, conditional on being matched, the daily probability that a firm hires a worker is 0.20. This is simply $(1 - \bar{p})$ in daily terms. Then, the daily probability that a firm meets a worker is 0.05/0.20 = 0.25. On a weekly basis, this is equivalent to setting $q = 0.8665$. Using the calibrated values of $\theta$ and $q$, I find $\zeta = 1.6783$.

There are two parameters in the selection cost function to be calibrated: $c_s$ and $c_p$. They determine how the quantity and quality margins are related. Hence, I choose these parameters value to match the average job filling rate and the average firm size. The job filling probability is calculated as 0.3017 in the previous paragraph. The average firm size

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\footnotesize


13 Consistently with the discussion in Mortensen and Nagypal (2007), a smaller value of $b$ attenuates the responses of labor market outcomes to aggregate productivity shocks.
in Business and Employment Dynamics (BED) is 21.6. I choose $c_s$ and $c_p$ to match these figures.

The last parameter to be calibrated is the fixed entry cost, $c_e$. I choose this value so that the expected value of an entrant is equal to zero in equilibrium.

## 5 Results

In this section, I present the results from the worker selection model regarding the cross sectional behavior of the hires-to-vacancy ratio. In their analysis, DFH construct a daily accounting model of establishment-level hiring dynamics and report estimates for the daily job filling rate, the theoretical counterpart of the hires-to-vacancy ratio. For comparability, I report the results for daily job filling rate from the worker selection model. My main finding is that the worker selection model accounts for about 30% of the variation of in the daily job filling rate observed in the data.

![Figure 2: The cross sectional relationship between growth rate and daily job filling rate.](image)

Figure 2 plots the daily job filling rate against the monthly employment growth rate bins from the worker selection model. The job-filling rate near zero percent growth is around

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$^{14}$I construct the growth rate bins so that the share of vacancies are equal in each bin. This procedure
2% and reaches 5% as the growth rate becomes about 5%. After this point, the response of the job-filling rate to employment growth becomes weaker, reaching only 6% at a 20% employment growth rate. The corresponding figures in the data are stronger: the job-filling rate is about 18% at a 20% employment growth rate.

There are several possible explanations for this gap between the model and the data. First, there can be micro level randomness in the data. Some firms are lucky to find good candidates and therefore fill vacancies at a higher rate and grow faster. The maximum value that the daily job filling rate can take in the model is 25%, which happens when the hiring standard is set equal to zero. This natural bound constrains the firms from achieving a higher job-filling rate. Second, there might be increasing returns at the establishment level. For example, it may be easier to attract more workers when the firm has more open positions. Hence, firms that are posting more vacancies meet proportionally more workers. Such a feature is absent in the model. Finally, there might be other margins, e.g. wage posting and firms’ search effort, that are not modeled in this paper. DFH provides an extensive discussion about these explanations.

To quantitatively evaluate the model, I calculate the elasticity of the daily job-filling rate with respect to the hires rate. The elasticity I calculated from the model is 0.24. From the data, DFH estimate this elasticity to be 0.82. These numbers imply that the model alone can account for about 30% of the variation in the growth rates. Further, DFH find that 0.04 of the 0.82 is due to increasing returns at the establishment level. In a simulation exercise, they also find that micro level randomness accounts for about 10% of the variation across the growth rate bins. These together imply that the worker selection can account for about 35% of the elasticity after controlling for scale and luck effects.

Figure 3 shows the relationship between the daily job filling rate and log firm size. Firm size is calculated as the average of the employment at the firm at the beginning and the end of the period. Since I do not directly target the firm size distribution, the firm sizes from the model are smaller than the firm sizes observed in the data. To make the size groups comparable to the data, I construct firm size bins such that log difference of average size in two consecutive bins are equal to those used in DFH. The daily job filling rate follows a hump-shaped pattern across the firm size bins, which is also present in the data. In the model, the job filling rate is about 6.5% at small firms, and decreases to 4.5% at medium and large firm sizes. The job-filling rate in the data goes from 6.6% down to 2.6% when moving from small establishments to large establishments. The job filling rate at large firms in the model stays high compared to the data. \(^{15}\)

Finally, Figure 4 plots the daily job filling rate against monthly worker turnover quin-
Figure 3: The cross sectional relationship between firm size and daily job filling rate.

Moving from low worker turnover rates to high turnover rates, the job filling rate rises from 1.5% to 8.0%. Similar numbers are present in the data as well, though the job-filling rate in the fifth quintile shoots up to 11.4% in the data.

6 Policy Analysis

Using the calibrated model in Section 4, I examine the effects of a hiring subsidy and a firing tax on aggregate and firm-level employment decisions. A hiring subsidy is a one-time payment made to firms for each worker they hire. A firing tax is a one-time payment collected from firms for each worker they fire. To highlight the effects of worker selection, I calibrate the standard DMP model and compare the results from labor market policy changes to those from the worker selection model. To ensure that the two models are comparable, I modify the standard DMP model, which I describe next.

\[ \text{growth rate becomes larger.} \] DFH constructs the growth rate bins in a similar fashion.

\[ \text{The difference might reflect that large firms have cost advantages in recruiting new workers, e.g. an advanced human resources department.} \] Introducing size dependent adjustment costs reduces the job filling rate at large firms, but increases it at small firms relative to the calculations in Figure 3.

\[ \text{The plot in Figure 4 does not start from 0%, which shows that about 18% of the firms do not hire any worker in a given month.} \]
6.1 Calibrating the Standard DMP Model

As in the worker selection model, the standard DMP model I describe in this section assumes decreasing returns to scale production technology and allows firms to hire multiple workers. I calibrate the model in a way that it matches the same targeted moments with the worker selection model described in Section 4. However, the two models differ regarding the cross-sectional variation in the hires-to-vacancy ratio. Although both models target the same average job filling probability, the standard DMP model implies a constant hires-to-vacancy ratio at the firm level, because firms are not allowed to screen workers. This allows me to isolate the effects of worker selection.

In the worker selection model, firms contact workers with probability $q$ but hire them with a smaller probability equal to $(1 - p)q$. Firms optimally choose $p$, which is allowed to vary across firms. In the standard DMP model, the contact probability coincides with the job filling probability, because workers are identical and firms indiscriminately hire all of the workers they contact. To create a gap between the contact and the job filling probabilities as in the worker selection model, I introduce an exogenous parameter, $\bar{p}$, to the standard DMP model. I set the value of $\bar{p}$ equal to the average value of the hiring standard in the
calibrated worker selection model so that the job filling probability becomes \((1 - \bar{p})q\) in the standard DMP model. This modification makes the average job filling probability the same between the two models, but the firm-level hires-to-vacancy ratio varies only in the worker selection model.

The choice of hiring standard also affects the probability of separation in the next period through the value of \(\gamma\). Define a new parameter \(p_\gamma\) in the standard DMP model such that the law of motion becomes:

\[
n' = (1 - \lambda)n + (1 - \bar{p})qv(1 - p_\gamma) \tag{24}
\]

\(p_\gamma\) now determines a common separation probability for newly hired workers. Comparing equation (24) and equation (4), \(1 - p_\gamma\) in the standard DMP model corresponds to \(\frac{1 - p^2}{\gamma(1 - p)}\) in the worker selection model. Recall that I targeted the job turnover rate from JOLTS in Section 4 to calibrate \(\gamma\). Similarly, I choose \(p_\gamma\) in the standard DMP model to match this target. While I search for the value of \(p_\gamma\), I also change the value of \(c_s, A, \sigma\) and \(c_e\). As in Section 4, \(c_s\) targets average firm size; \(A\) targets an equilibrium value of \(\Omega\) equal to 1, \(\sigma\) is set to match the hires rate, and \(c_e\) satisfies the free entry condition. I maintain the restriction that \(\frac{b}{\sqrt{N}} = 0.72\). I also drop \(c_p\) from the selection cost function while preserving its quadratic form in the number of applicants. I set the value of \(c_v\) equal to the value obtained from the calibration of worker selection model. The precise treatment of this parameter does not change the conclusions of this section. Finally, I set all the remaining parameters equal to their corresponding values in Table 1. The values of the newly calibrated parameters are presented in Table 2.

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<th>Parameter</th>
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<td>(c_e)</td>
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Table 2: Calibrated Parameters of the standard DMP model (Weekly)
6.2 Wage Bargaining with a Hiring Subsidy and a Firing Tax

A hiring subsidy and a firing tax affect the wage bargaining outcome because these policy instruments affect the surplus to the firm and the workers. Let \( s \) and \( \tau \) denote a hiring subsidy and a firing tax, respectively, measured in terms of the consumption good. Then, the Stole-Zwiebel bargaining rules defined in equations (13) and (14) become

\[
\phi(D_n(\tilde{n}, r, p, \varepsilon) + \tau) = (1 - \phi)(V^n(n', \varepsilon) - V^n) \tag{25}
\]

\[
\phi(D_r(\tilde{n}, r, p, \varepsilon) + s) = (1 - \phi)(V^p(n', \varepsilon) - V^u) \tag{26}
\]

When a firm is paid \( s \) for hiring a new worker, the surplus to the firm from the newly hired worker increases by \( s \). When a firm is forced to pay a firing tax of \( \tau \), the surplus to the firm from keeping an existing worker increases by \( \tau \) as the firm avoids paying the firing tax. Following the same steps in the derivation of wages detailed in Appendix A, I show that the wage equations for existing and new workers become:

\[
w^p(n', \varepsilon, p) = g(p) \frac{\alpha \phi}{1 - \phi + \alpha \phi} A\varepsilon n'^{a-1} + (1 - \phi)\Omega + \phi s \tag{27}
\]

where \( \Omega \) is defined as in equation (17). One can obtain the wage equation for the standard DMP model after replacing \( g(p) \) with \( 1 - p \gamma \) in the equations above.

6.3 Effects of a Hiring Subsidy

I calculate the response of labor market outcomes to incremental increases in hiring subsidy.\(^\text{17}\) Table 3 reports equilibrium labor market outcomes and total output net of adjustment cost from the worker selection and the standard DMP models. I also report the amount of subsidy as a fraction of the average wage of a newly hired worker for each model to compare the relative size of the subsidy between the two models.

If a policymaker assesses the hiring subsidy using the standard DMP model, he will not be optimistic about the hiring subsidy in combating unemployment. When firms are subsidized about half of the average wage of newly hired worker, the decline in unemployment is only 0.08 percentage points. On the other hand, the worker selection model predicts that the same policy is a powerful tool to reduce unemployment. A hiring subsidy that is equal to the half of the average wage of a newly hired worker reduces the unemployment rate by 0.5 percentage points.

The aggregate hires-to-vacancy ratio responses are different between the two models. In 17I assume that government finances the hiring subsidy through a lump-sum tax on workers.
Table 3: The Effects of Hiring Subsidy on Equilibrium

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<td>% of average $w^P$</td>
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<tr>
<td>Unemployment Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>5.35%</td>
<td>5.25%</td>
<td>5.12%</td>
<td>4.98%</td>
<td>4.87%</td>
<td>4.79%</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>5.35%</td>
<td>5.34%</td>
<td>5.32%</td>
<td>5.31%</td>
<td>5.29%</td>
<td>5.28%</td>
</tr>
<tr>
<td>Hires-to-vacancy ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.302</td>
<td>0.311</td>
<td>0.321</td>
<td>0.331</td>
<td>0.337</td>
<td>0.325</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.302</td>
<td>0.301</td>
<td>0.300</td>
<td>0.299</td>
<td>0.297</td>
<td>0.295</td>
</tr>
<tr>
<td>Contact Prob(q)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.867</td>
<td>0.863</td>
<td>0.858</td>
<td>0.851</td>
<td>0.840</td>
<td>0.802</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.867</td>
<td>0.863</td>
<td>0.861</td>
<td>0.858</td>
<td>0.854</td>
<td>0.848</td>
</tr>
<tr>
<td>Hiring Standard(p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.652</td>
<td>0.640</td>
<td>0.626</td>
<td>0.612</td>
<td>0.599</td>
<td>0.595</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.652</td>
<td>0.652</td>
<td>0.652</td>
<td>0.652</td>
<td>0.652</td>
<td>0.652</td>
</tr>
<tr>
<td>Net Output (% change)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.000%</td>
<td>0.020%</td>
<td>0.047%</td>
<td>0.067%</td>
<td>0.084%</td>
<td>0.068%</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.000%</td>
<td>0.019%</td>
<td>0.031%</td>
<td>0.040%</td>
<td>0.049%</td>
<td>0.053%</td>
</tr>
</tbody>
</table>

the standard DMP model, the aggregate hires-to-vacancy ratio decreases with the hiring subsidy. Because the aggregate level of vacancies increases with the subsidy, the probability that a firm contacts a worker goes down due to increased market tightness. If firms can select workers, however, there is an additional effect on the hires-to-vacancy ratio through the optimal choice of the hiring standard. When firms are subsidized for hiring new workers, they become less picky about the workers as they are compensated for the loss due to hiring an unproductive worker. The combined effect on the hiring vacancy-ratio is ambiguous. Table 3 shows that the effect of the latter is greater than the former when the subsidy is small. When the subsidy becomes large, the hires-to-vacancy ratio starts to decline.

The output net of adjustment costs initially increases with the hiring subsidy, but the increases in net output is less than 1% in each case. When the subsidy exceeds 0.4, output
net of the adjustment costs starts declining in both models.

![Graph showing the difference in the cumulative distribution of monthly employment growth rates in response to a hiring subsidy equal to 0.1 units of consumption good.](image)

**Figure 5:** Difference in the cumulative distribution of monthly employment growth rates in response to a hiring subsidy equal to 0.1 units of consumption good.

The effects of a hiring subsidy on employment growth rates in the cross section are qualitatively different in the worker selection and the standard DMP models. To highlight the impact of a hiring subsidy on employment growth rates, I divide employment growth rates into non-overlapping bins with the first bin including all of the shrinking firms. Since the firms are concentrated in low employment growth rate bins, I construct the growth rate bins so that the bins become progressively wider. I calculate monthly employment growth rates for each firm and place them into their corresponding employment growth rate bins. Then, I introduce a hiring subsidy equal to 0.1 and calculate the monthly employment growth rates. When I place the firms into their corresponding employment growth rate bins, I use the weights in the steady state stationary distribution without the subsidy. Finally, I calculate the cumulative distribution with and without the hiring subsidy for each model and plot the difference in Figure 5.

The cumulative distribution after the hiring subsidy evaluated at each employment growth rate is smaller than the corresponding value in the employment growth distribution without the subsidy for both models. A negative value in Figure 5 at all growth rates implies that the entire distribution of the employment growth shifted to the right. However, the effects on individual firms are different in the worker selection and the standard DMP models. The number of firms around 0% employment growth decreases sharply in each model. The decline in the standard DMP model is larger, but becomes very quickly close to
zero. This pattern implies that the hiring subsidy increases employment growth at slowly growing firms, but leaves the right tail of the distribution unaltered. In contrast, the decline in the number of firms around 0% employment growth rate is smaller in the worker selection, but approaches zero slowly relative to the standard DMP model. For the standard DMP model, Figure 5 implies that the increase in the number of firms with employment growth rate between 1% and 8% is roughly three times larger than the increase in the number of firms with more than 8% employment growth rate. The opposite is true for the worker selection model. In other words, most of the shift in the worker selection model occurs in the right tail of the employment growth distribution.

The difference between the two models stems from firms’ ability to change the hiring standard in response to a hiring subsidy in the worker selection model. A hiring subsidy shifts the marginal cost of labor adjustment in a parallel fashion if firms cannot change the hiring standard. Because the production function is Cobb-Douglas, this induces a larger increase in employment at larger firms, which tend to have lower employment growth rates. However, when firms can select workers, small and growing firms lower their hiring standards in response to a hiring subsidy. The ability to select workers reduces the marginal cost of labor adjustment more at these firms. Hence, small and growing firms increase their employment more compared to large firms. The stronger response of the slow and growing firms causes the employment growth distribution to shift more in the right tail of the distribution.

Empirical evidence from the literature supports the predictions of the worker selection model. Perloff and Wachter (1979) analyze the effects of the New Jobs Tax Credit of the stimulus package in 1977 on employment growth. Despite its complex structure, the tax credit program affects incremental hirings rather than total employment, and hence similar to the hiring subsidy experiment in this section. Perloff and Wachter (1979) use a follow-up survey to the subsidy program which enables them to distinguish firms that knew about the program from those who did not. They use this knowledge information to identify the effects of the subsidy on employment growth. According to their findings, the knowledge about the tax credit program shifts the employment growth distribution to the right and most of this shift occurs in the right tail of the distribution.

### 6.4 The Effects of a Firing Tax

Compared to a hiring subsidy, a firing tax has opposite effects on the equilibrium outcomes in both models, but the differences between the two models remain. The results from incremental increases in the firing tax are presented in Table 4 for worker selection and the
Table 4: The Effects of a Firing Tax on Equilibrium

<table>
<thead>
<tr>
<th>Firing Tax Levels in Consumption Good</th>
<th>0.000</th>
<th>0.100</th>
<th>0.300</th>
<th>0.500</th>
<th>0.700</th>
<th>0.900</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of average $w^p$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.00%</td>
<td>12.84%</td>
<td>43.44%</td>
<td>83.22%</td>
<td>137.26%</td>
<td>215.21%</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.00%</td>
<td>12.97%</td>
<td>44.64%</td>
<td>87.23%</td>
<td>147.54%</td>
<td>239.63%</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>5.35%</td>
<td>5.47%</td>
<td>5.70%</td>
<td>5.91%</td>
<td>6.12%</td>
<td>6.33%</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>5.35%</td>
<td>5.36%</td>
<td>5.39%</td>
<td>5.42%</td>
<td>5.45%</td>
<td>5.48%</td>
</tr>
<tr>
<td>Hires-to-vacancy ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.302</td>
<td>0.294</td>
<td>0.280</td>
<td>0.269</td>
<td>0.258</td>
<td>0.249</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.302</td>
<td>0.302</td>
<td>0.304</td>
<td>0.306</td>
<td>0.307</td>
<td>0.309</td>
</tr>
<tr>
<td>Contact Prob(q)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.867</td>
<td>0.871</td>
<td>0.878</td>
<td>0.884</td>
<td>0.889</td>
<td>0.895</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.867</td>
<td>0.863</td>
<td>0.861</td>
<td>0.858</td>
<td>0.854</td>
<td>0.848</td>
</tr>
<tr>
<td>Hiring Standard(p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.652</td>
<td>0.663</td>
<td>0.681</td>
<td>0.697</td>
<td>0.710</td>
<td>0.722</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.652</td>
<td>0.652</td>
<td>0.652</td>
<td>0.652</td>
<td>0.652</td>
<td>0.652</td>
</tr>
<tr>
<td>Net Output (% change)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker Selection</td>
<td>0.000%</td>
<td>-0.050%</td>
<td>-0.146%</td>
<td>-0.248%</td>
<td>-0.351%</td>
<td>-0.456%</td>
</tr>
<tr>
<td>Standard DMP</td>
<td>0.000%</td>
<td>-0.025%</td>
<td>-0.089%</td>
<td>-0.157%</td>
<td>-0.228%</td>
<td>-0.301%</td>
</tr>
</tbody>
</table>

standard DMP models.\(^{18}\)

An increase in the firing tax unambiguously increases the unemployment rate in both models, because the equilibrium mass of active firms decreases due to increased labor adjustment costs. However, the increase in the unemployment rate in the worker selection model is about ten times larger.

The aggregate hires-to-vacancy ratio slightly increases in the standard DMP model, but declines in the worker selection model. Even when the firing tax doubles the average weekly wage of a newly hired worker, the increase in the hiring standard surpasses the increase in the aggregate contact probability. The dominant effect of the hiring standard reduces the aggregate hires-to-vacancy ratio.

\(^{18}\)I assume collected tax is distributed to workers as a lump-sum transfer.
Finally, net output decreases with the firing tax. There is not a simple Hosios condition that guarantees that the competitive equilibrium is socially optimal, but Tables 3 and 4 imply that subsidizing new hires is welfare improving over taxing firings. The reason for this outcome is that there are too few active firms in the competitive equilibrium and some workers are inefficiently employed at very large firms. A hiring subsidy increases net output not only by encouraging new firm entry, but also by causing very large firms to shed marginal workers who could be more efficiently employed at smaller firms.

Figure 6: Change in the worker turnover rates in response to a firing tax equal to 0.9 units of consumption goods across worker turnover rate bins.

The effects of a firing tax on worker turnover rates at the firm level are qualitatively different in the worker selection and the standard DMP models. To show the effects of a firing tax on worker turnover rates in the cross section, I divide the worker turnover rates into non-overlapping bins. As in the previous section, I start with narrower bins and make them progressively wider. First, I calculate monthly worker turnover rates for each firm without the firing tax and place them into their corresponding worker turnover bin. Then, I impose a tax of 0.1 in terms of the consumption good on firings and find the new monthly worker turnover rate for every firm. Finally, I calculate the change in the average worker turnover rate in each bin using the weights in the stationary distribution of the pre-policy steady state. I plot the difference in Figure 6. The left panel corresponds to the worker selection model and the right panel corresponds to the standard DMP model.

From Figure 6, the decrease in the worker turnover rate after a firing tax is larger at firms with initially higher worker turnover rate in the worker selection model. In contrast, the
decrease in the worker turnover rate is larger at firms with initially lower worker turnover rates in the standard DMP model. The reason for the difference between the models is that a firing tax discourages firings at shrinking firms in the standard DMP model, but it has a relatively small effect on worker turnover rates in expanding firms. In the stationary distribution of the standard DMP model, shrinking firms are concentrated at low to medium worker turnover rate bins. In contrast, a firing tax reduces worker turnover rate relatively more at growing firms in the worker selection model. In response to a firing tax, growing firms increase their hiring standards to avoid paying a firing tax for a potentially low quality match. Therefore, worker turnover rates decline relatively more at high worker turnover firm in the worker selection model.

The predictions of the worker selection model is consistent with empirical evidence from the literature. Haltiwanger, Scarpetta and Schweiger (2010) use a harmonized data on job creation and job destruction from emerging and developed countries and estimate the effects of hiring and firing regulations on job reallocation rates. Using a difference-in-difference approach, they find that firms in the industries and size classes that require more frequent employment changes, e.g., technological changes, are affected relatively more from hiring and firing restrictions. The graphs in Figure 6 confirms that the predictions of the worker selection model is consistent with the data.

7 Conclusion

In the U.S., the hires-to-vacancy ratio in the cross section (i) rises steeply with employment growth rate, (ii) declines with firm size, and (iii) increases with worker turnover rate. These patterns of the hires-to-vacancy ratio are incompatible with the standard DMP model. The reason for the failure of the standard DMP model is due to the use of an aggregate matching function, which postulates that all of the firms fill vacancies at a common rate. Even extensions to the standard DMP model that allow firms to hire multiple workers fail to generate the cross-sectional variation in the hires-to-vacancy ratio due to the use of an aggregate matching function.

I extend the standard DMP model to allow firms to selectively hire multiple workers among a pool of applicants to account for the firm-level behavior of the hires-to-vacancy ratio. I motivate selection of workers by introducing match-specific quality shocks to the model, which determine the productivity of a worker at the hiring firm and can only be partially observed at the time of hiring. Firms recruit, screen, and interview applicants to make inference about the match-quality of the potential hires. I model this selection mechanism by allowing firms to choose a minimum quality threshold below which applicants are not
hired. Firms can fill vacancies at different rates by adjusting their hiring standards and this mechanism generates cross-sectional variation in the hires-to-vacancy ratio are consistent with the data. Calibrated to the salient features of the U.S. labor market, the worker selection model accounts for about 30% of the variation in the hires-to-vacancy ratio across different growth rates. The remaining part can be explained by micro-level randomness, increasing returns to scale at establishment level, other mechanisms such as wage posting and firms’ search effort.

In the policy analysis section, I analyze the effects of a hiring subsidy and a firing tax on labor market outcomes. The worker selection and the standard DMP models have different policy implications at both the aggregate and firm levels. The standard DMP model predicts that a hiring subsidy would reduce unemployment only slightly. However, the decline in the worker selection model is substantial: the unemployment rate would go down by half of a percentage points if firms are subsidized for half of the wages of a newly hired worker.

The worker selection model also gives new insights about how different firm groups are affected by labor market policies. The worker selection model implies that labor market policies have a bigger impact on employment dynamics at fast growing and high worker turnover firms, while the standard DMP model implies the opposite. Empirical evidence from the literature supports the predictions of the worker selection model.
8 Appendix

A Derivation of Wage Functions

The derivation of the wage functions exploits the fact that the continuation values for the firm and the workers cancel each other from the first order condition for \( n' \) and the envelope condition. Let \( J(n', \varepsilon) = E_{\varepsilon'|\varepsilon} \Pi_{n'}(n', \varepsilon') \). From (12), the marginal value of an existing and potential worker are:

\[
D_{\bar{n}}(.) = \alpha A \varepsilon n^{\alpha - 1} - (w^n)'(.) \bar{n} - w^n(.) - (w^p)'(.) r + J(.) \tag{28}
\]

\[
D_r(.) = g(p) \alpha A \varepsilon n^{\alpha - 1} - g(p)(w^n)'(.) \bar{n} - g(p)(w^p)'(.) r - w^p(.) + g(p) J(.) \tag{29}
\]

where \((w^n)'(.)\) and \((w^p)'(.)\) are the derivatives of the wage functions with respect to \( n' \). Rearranging the bargaining solution in (13), I get:

\[
(1 - \phi)(w^n(.) - \Omega + \beta(1 - \delta)(1 - \lambda)[E_{\varepsilon'|\varepsilon} V^n(n', \varepsilon') - V^n])
= \phi(\alpha A \varepsilon n^{\alpha - 1} - (w^n)'(.) \bar{n} - w^n(.) - (w^p)'(.) r + J(.)) \tag{30}
\]

where \( \Omega \) is defined in (17). First, I show that \( J(n', \varepsilon) = \beta(1 - \delta)(1 - \lambda)[E_{\varepsilon'|\varepsilon} V^n(n', \varepsilon') - V^n] \).

To see that, re-write the dynamic problem of a hiring firm before inserting the wage functions and after replacing the constraint in 4:

\[
\Pi^h(n, \varepsilon) = \max_{n', \varepsilon' \in [0, 1]} - \frac{c_v}{q} \frac{\gamma (n' - (1 - \lambda)n)}{1 - p^{\gamma}} - \frac{c_s}{2} \exp \left( \frac{c_p}{2} \frac{p^2}{1 - p^{\gamma}} \right) \left( \frac{\gamma (n' - (1 - \lambda)n)}{1 - p^{\gamma}} \right)^2
+ A \varepsilon n^{\alpha} - (1 - \lambda)n w^n(n', \varepsilon') - \frac{(1 - p)\gamma (n' - (1 - \lambda)n)}{1 - p^{\gamma}} w^p(n', \varepsilon')
+ \beta(1 - \delta) E_{\varepsilon'|\varepsilon} \Pi(n', \varepsilon') \tag{31}
\]

The first order condition for \( n' \) is:

\[
- \frac{c_v}{q} \frac{\gamma}{1 - p^{\gamma}} - c_s \exp \left( \frac{c_p}{2} \frac{p^2}{1 - p^{\gamma}} \right) \left( \frac{\gamma}{1 - p^{\gamma}} \right) (n' - (1 - \lambda)n)
+ \alpha A \varepsilon n^{\alpha - 1} - (1 - \lambda)n(w^n)' - \frac{(1 - p)\gamma}{1 - p^{\gamma}} w^n(.) - \frac{(1 - p)\gamma (n' - (1 - \lambda)n)}{1 - p^{\gamma}} (w^p)'(.)
+ \beta(1 - \delta) E_{\varepsilon'|\varepsilon} \Pi_{n'}(n', \varepsilon') = 0 \tag{32}
\]
Next, conditional on hiring, the envelope condition implies:

\[
\Pi_{n'}(n', \varepsilon) = (1 - \lambda) \left( \frac{c_v}{q} \frac{\gamma}{1 - p^{\gamma}} + c_s \exp \left( \frac{c_p}{2} p^2 \right) \left( \frac{\gamma}{1 - p^{\gamma}} \right)^2 (n' - (1 - \lambda)n) \right) \\
+ (1 - \lambda) \left( -w_n(.) + \frac{(1 - p)\gamma}{1 - p^{\gamma}} w_p(.) \right)
\]  

(33)

Replacing the first line in (33) from (32) and substituting for \( \tilde{n} \) and \( r \), one obtains:

\[
\Pi_{n'}(n', \varepsilon) = (1 - \lambda) D_{\tilde{n}}(.)
\]  

(34)

If the firm is neither hiring nor firing, (34) holds by definition. Finally, if the firm is firing workers, then the marginal surplus of an existing worker is equal to 0. Equivalently, \( \Pi_{n'}(n', \varepsilon) = 0 \). Hence, (34) is trivially satisfied. From the definition of \( J(.) \) and using (34), one gets \( J(n) = \beta(1 - \delta)(1 - \lambda) D_{\tilde{n}}(.) \), which further implies \( J(n', \varepsilon) = \beta(1 - \delta)(1 - \lambda)[E_{\varepsilon'[\varepsilon] V^{n}(n', \varepsilon') - V^n}] \) by (13).

The bargaining equations now can be written as follows:

\[
\phi \left( \alpha A \varepsilon n'^{\alpha - 1} - (w_n)'(.)\tilde{n} - (w_p)'(.)r \right) = (1 - \phi)(w^n(.) - \Omega)
\]  

(35)

\[
g(p)\phi \left( \alpha A \varepsilon n'^{\alpha - 1} - (w_n)'(.)\tilde{n} - (w_p)'(.)r \right) = (1 - \phi)(w^n(.) - \Omega)
\]  

(36)

Multiplying (35) by \( g(p) \) and subtracting from (36) implies:

\[
(w^p)(.) = g(p)(w^n)(.) + (1 - g(p))\Omega
\]  

(37)

After taking the derivative with respective to \( n' \) and plugging this back in (28), I obtain the following first order differential equation in \( n' \):

\[
w^n(.) + \phi n'(w^n)'(.) = \phi \alpha A \varepsilon n'^{\alpha - 1} + (1 - \phi)\Omega
\]  

(38)

The solution to this differential equation is given by (27). The constant of integration is set to zero so that \( n'w(.) \to 0 \) as \( n' \to 0 \). The wage equation for newly hired workers can be obtained from (37).
B Recursive Stationary Equilibrium

Definition 1. (Recursive Stationary Equilibrium) The recursive stationary equilibrium consists of value function for firms, \( \Pi(n, \varepsilon) \); a set of decision rules for vacancies, hiring standard, firings and employment, \( v(n, \varepsilon) \), \( p(n, \varepsilon) \), \( f(n, \varepsilon) \) and \( n'(n, \varepsilon) \); value functions for workers, \( V^n(n, \varepsilon) \) and \( V^p(n, \varepsilon) \); wage functions, \( w^n(n', \varepsilon) \) and \( w^p(n', \varepsilon, p) \); market tightness and aggregate matching probability, \( \theta \) and \( q \); value of unemployment at the beginning and bargaining stages, \( V^u \) and \( V^u \); and a stationary distribution firms across productivities and employment, \( \Gamma(n, \varepsilon) \), such that:

1. \( \theta \) and \( q \) are related according to (2).
2. Firm’s Optimization: Given \( q \), \( w^n(n', \varepsilon) \) and \( w^p(n', \varepsilon, p) \), the set of decision rules, \( v(n, \varepsilon) \), \( p(n, \varepsilon) \), \( f(n, \varepsilon) \) and \( n'(n, \varepsilon) \), solve firms’ problem described by equations (4), (5), (6) and (7).
3. Worker Value Functions: Given \( \theta q \), \( w^n(n', \varepsilon) \), \( w^p(n', \varepsilon, p) \) and firms' decision rules, \( v(n, \varepsilon) \), \( p(n, \varepsilon) \), and \( n'(n, \varepsilon) \), value functions for workers, \( V^n(n, \varepsilon) \), \( V^p(n, \varepsilon) \), \( V^u \) and \( V^u \), satisfy equations (8), (9), (10) and (11).
4. Wage Bargaining: The wage equations, \( w^n(n', \varepsilon) \) and \( w^p(n', \varepsilon, p) \), satisfy (12), (13) and (14).
5. Free-entry condition in (19) holds.
6. Consistency: The stationary distribution \( \Gamma(n, \varepsilon) \) is consistent with the firm’s decision rules and satisfies (18).

C Properties of the Adjustment Cost Function

As in the text, let \( \Delta = (n' - (1 - \lambda)n) \). We are seeking the optimal choice for \( p \) given \( \Delta \) to minimize total cost of adjustment. For brevity, let \( f(p) = \frac{1}{1 - p} \) and \( h(p) = c_p \exp \left( \frac{c_p}{2} p^2 \right) (f(p))^2. \)

Note that the natural logarithm of \( h(p) \), \( f(p)^2 \) and \( c_p \exp(p^2) \) are all convex. I use this observation to show the convexity of \( C(\Delta) \). The minimization problem of the firm is:

\[
C(\Delta) = \min_{p \in [0,1]} \gamma \Delta \left( (\Omega + c_v / q) f(p) - \Omega f(p) p + h(p) \Delta \right)
\]

Let \( \Upsilon(p, \Delta) \) denote the objective function of the problem above. I have already established in the text that the solution is interior and unique. Then, the first order condition (FOC)
and the second order condition (SOC) are:

\[ \psi_{p}(p, \Delta) = 0 \]
\[ \psi_{pp}(p, \Delta) > 0 \]

Totally differentiating the FOC, one obtains:

\[ p'(\Delta) = \frac{dp}{d\Delta} = -\frac{\psi_{p\Delta}}{\psi_{pp}} = -\frac{h'(p)\Delta}{\psi_{pp}} \leq 0 \]

The last inequality follows from the SOC. Further, the cost function satisfies:

\[ C(\Delta) = \psi(p(\Delta), \Delta) \]

Taking the derivative with respect to \( \Delta \):

\[ C'(\Delta) = \psi_{p}p'(\Delta) + \psi_{\Delta} \]

By the FOC, the first term is zero. Hence:

\[ C'(\Delta) = (\Omega(1 - p) + c_v/q)f(p) + 2h(p)\Delta > 0 \]

Finally, differentiating the expression for \( C'(\Delta) \) yields:

\[ C''(\Delta) = \psi_{pp}p''(\Delta) + \psi_{p\Delta}p'(\Delta) + \psi_{\Delta\Delta} \]

By the FOC, the first term is zero. Rearranging the terms and replacing for \( p'(\Delta) \) yields:

\[ C''(\Delta) = \psi_{\Delta\Delta} - \frac{\psi_{p\Delta}}{\psi_{pp}} \]

By the SOC, the adjustment cost function is convex iff \( \psi_{\Delta\Delta} \psi_{pp} - \psi_{p\Delta}^2 \geq 0 \). This requires:

\[ \Delta(\Omega(1 - p) + c_v/q)f''(p) - 2\Omega f'(p))2h(p)\Delta \]
\[ + \Delta^2(\Delta^2 + 2h''(p)h(p) - \Delta^3(h'(p))^2 \geq 0 \]

The first term is positive from the definition of \( f(p) \). A sufficient condition for the second term to be positive is that \( h(p)h''(p) - (h'(p))^2 \geq 0 \). This condition is also satisfied by log-convexity of \( h(p) \).
References


