Abstract—Data access control is an effective way to ensure data security in the cloud. However, due to data outsourcing and untrusted cloud servers, the data access control becomes a challenging issue in cloud storage systems. Existing access control schemes are no longer applicable to cloud storage systems, because they either produce multiple encrypted copies of the same data or require a fully trusted cloud server. Ciphertext-Policy Attribute-based Encryption (CP-ABE) is a promising technique for access control of encrypted data. However, due to the inefficiency of decryption and revocation, existing CP-ABE schemes cannot be directly applied to construct data access control scheme for multi-authority cloud storage systems, where users may hold attributes from multiple authorities. In this paper, we propose DAC-MACS (Data Access Control for Multi-Authority Cloud Storage), an effective and secure data access control scheme with efficient decryption and revocation. Specifically, we construct a new multi-authority CP-ABE scheme with efficient decryption, and also design an efficient attribute revocation method that can achieve both forward security and backward security. We further propose an extensive data access control scheme (EDAC-MACS), which is secure under weaker security assumptions.

Index Terms—Access Control, CP-ABE, Decryption Outsourcing, Attribute Revocation, Multi-authority Cloud.

I. INTRODUCTION

Cloud storage is an important service of cloud computing [1]. It allows data owners to host their data in the cloud and rely on cloud servers to provide “24/7/365” data access to users (data consumers). Data access control is an effective way to ensure the data security in the cloud. However, due to the data outsourcing, the cloud server cannot be fully trusted to provide data access control service, which means existing server-based access control methods are no longer applicable to cloud storage systems. To achieve data access control on untrusted servers, traditional methods usually encrypt the data and only users holding valid keys are able to decrypt. Although these methods can provide secure data access control, the key management is very complicated when more users are in the system. Data owners also have to stay online all the time to deliver keys to new users. Moreover, for each data, there are multiple copies of ciphertexts for users with different keys, which will incur high storage overhead on the server.

Ciphertext-Policy Attribute-based Encryption (CP-ABE) [2]–[6] is regarded as one of the most suitable technologies for data access control in cloud storage systems, because it gives the data owner more direct control on access policies and does not require the data owner to distribute keys. In CP-ABE scheme, there is an authority that is responsible for attribute management and key distribution. The authority can be the registration office in a university, the human resource department in a company, etc. The data owner defines the access policies and encrypts data under the policies. Each user will be issued a secret key according to its attributes. A user can decrypt the ciphertexts only when its attributes satisfy the access policies.

In cloud storage systems, a user may hold attributes issued by multiple authorities and the owner may share data with users administrated to different authorities. For instance, in an E-health system, the medical data may be shared only with a user who has the attribute of “Doctor” issued by a hospital and the attribute “Medical Researcher” issued by a medical research center. Some CP-ABE schemes [7]–[10] have been proposed for such multi-authority systems. However, due to the inefficiency of computation, they cannot be directly applied to construct the data access control scheme. Basically, there are two operations in access control that require efficient computation, namely decryption and revocation.

Revocation Efficiency: Data access in cloud storage systems is not static, as as employees are hired/fired or promoted/demoted, it will be necessary to change the attributes of users. To guarantee the security of attribute revocation, there are two requirements: 1) Backward Security: The revoked user (whose attributes are revoked) cannot decrypt new ciphertexts that require the revoked attributes for decryption; 2) Forward Security: The newly joined users who have sufficient attributes are also able to decrypt the previously published ciphertexts. To achieve these two requirements, a trivial method is to re-encrypt all the data. But it incurs a high computation overhead as the amount of data is massive. This motivates us to develop a new method that can efficiently deal with the attribute revocation of users.

Decryption Efficiency: In CP-ABE systems, the users need to decrypt the data by using their secret keys. However, nowadays, users usually use their mobile devices (e.g., smart phones, tablets etc.) to access the cloud data, and the computation abilities of mobile devices are not as powerful as the one
of PCs. This motivates us to outsource the main computation of decryption into the cloud server, while still keep the data privacy against the cloud server.

In this paper, we first construct a new multi-authority CP-ABE scheme with efficient decryption and propose an efficient attribute revocation method for it. Then, we apply them to design an effective access control scheme for multi-authority cloud storage systems. The main contributions of this work can be summarized as follows.

1) We propose DAC-MACS (Data Access Control for Multi-Authority Cloud Storage), an effective and secure data access control scheme for multi-authority cloud storage systems, which is secure in the random oracle model and has better performance than existing schemes.
2) We construct a new multi-authority CP-ABE scheme with efficient decryption. Specifically, we outsource the main computation of the decryption by using a token-based decryption method.
3) We also design an efficient immediate attribute revocation method for multi-authority CP-ABE scheme that achieves both forward security and backward security. Moreover, it incurs less communication cost and computation cost during the attribute revocation.

Compared with the previous conference version [11], we highly improve the security of DAC-MACS and make it more practical for multi-authority cloud storage systems. Specifically, we mainly address the security weakness caused by the collusion between non-revoked users and the corrupted AA. We first give a straightforward solution by making a security assumption that all the non-revoked users will not send their received update keys to the revoked user. We further remove this assumption and propose an extensive data access control scheme (EDAC-MACS) that can achieve the same security goal. In EDAC-MACS, the revoked user will not be able to get illegal data access even with the help of any corrupted AA and non-revoked user. Without such assumption, EDAC-MACS is more practical to be implemented in multi-authority cloud storage systems. We also provide the security analysis of EDAC-MACS and prove that it is secure under weaker security assumptions.

The remaining of this paper is organized as follows. We first define the system model, framework and security model in Section II. Then, we propose a new multi-authority CP-ABE scheme with efficient decryption and revocation, and then apply it to construct DAC-MACS in Section III. In Section IV, we analyze DAC-MACS in terms of both the security and the performance. We further extended our DAC-MACS to be secure under weaker assumptions in Section V. Section VI gives the related work. Finally, the conclusion is given in Section VII and the detailed security proof is described in the Appendix.

II. SYSTEM MODEL AND SECURITY MODEL

A. System Model

We consider a cloud storage system with multiple authorities, as shown in Fig.1. The system model consists of five types of entities: a global certificate authority (CA), attribute authorities (AAs), cloud server (server), data owners (owners) and data consumers (users).

CA. The CA is a global trusted certificate authority in the system. It sets up the system and accepts the registration of all the users and AAs in the system. For each legal user, the CA assigns a global unique user identity to it and also generates a global secret/public key pair for this user. However, the CA is not involved in any attribute management and any generation of secret keys that are associated with attributes.

AA. Every AA is an independent attribute authority that is responsible for issuing, revoking and updating user’s attributes according to its role or identity. Each AA is responsible for generating a public attribute key for each attribute it manages and a secret key for each user reflecting their attributes.

Server. The cloud server stores owners’ data and provides data access service to users. It also helps users decrypt ciphertexts by generating decryption tokens and helps owners update ciphertexts when an attribute revocation happens.

Owners. Before outsourcing the data, each owner first encrypts the data with content keys by using symmetric encryption techniques. Then, the owner defines the access policies over attributes from multiple AAs and encrypts content keys under the policies. They do not trust on the server to do data access control. Instead, they assume that the server may give the data to all the users in the system. But, the access control happens inside the cryptography. That is only when the user’s attributes satisfy the access policy defined in the ciphertext, the user is able to decrypt the ciphertext.

Users. Each user is assigned with a global user identity from the CA and can freely query ciphertexts from the server. To decrypt a ciphertext, each user may submit their secret keys issued by some AAs together with its global public key to the server and ask for a decryption token. The user then uses the received decryption token to decrypt the ciphertext along with its global secret key. Only when the user’s attributes satisfy the access policy defined in the ciphertext, the server can generate the correct decryption token. The secret keys and the global user’s public key can be stored on the server; subsequently, the user does not need to submit any secret keys if no secret keys are updated for further decryption token generation.
B. Framework

**Definition 1** (DAC-MACS). The framework of DAC-MACS contains the following phases:

**Phase 1: System Initialization**

This phase consists of the following algorithms:

- **CASetup**(\(1^k\)) \(\rightarrow\) (MSK, SP, (skCA, vkCA)). The CA setup algorithm takes no input other than the implicit security parameter \(k\). It outputs the master key MSK, the system parameter SP, a pair of signature and verification key \((skCA, vkCA)\) of the CA.
- **UserReg**(SP, skCA, Info\(_x\)) \(\rightarrow\) (uid, GPK\(_{uid}\), GSK\(_{uid}\), Cert\(_{uid}\)). The user registration algorithm takes the system parameter SP, the CA’s signature key skCA and the user information Info\(_x\) (e.g., name, birthday etc.) as inputs. It authenticates the user and assigns a global unique user identity uid to the user. It outputs the user identity uid, a pair of global public/secret key \((GPK\(_{uid}\), GSK\(_{uid}\))\) and a certificate Cert\(_{uid}\) which is signed by the CA.
- **AAReg**\(\rightarrow\) (aid). The attribute authority registration algorithm takes the information of an attribute authority Info\(_{AA}\) as input. It authenticates the AA and outputs a global authority identity aid for this AA.
- **AASetup**(SP, aid) \(\rightarrow\) (SK\(_{aid}\), PK\(_{aid}\), \(\{VK\(_{uid}\), PK\(_{uid}\}\)\). The attribute authority setup algorithm takes the system parameter SP and the global authority identity aid as inputs. It outputs a pair of secret/public authority key \((SK_{aid}, PK_{aid})\), the set of version keys and public attribute keys \(\{VK_{uid}, PK_{uid}\}\) for each attributes \(x\).

**Phase 2: Secret Key Generation**

- **SKKeyGen**(SK\(_{uid}\), SP, \{PK\(_{uid}\), S\(_{aid,uid}\), Cert\(_{uid}\)\) \(\rightarrow\) SK\(_{uid,auid}\). The secret key generation algorithm takes as inputs the secret authority key SK\(_{aid}\), the system parameter SP, the set of public attribute keys \(\{PK_{uid}\}\), a set of attributes S\(_{aid,uid}\) that describes the secret key, and the certificate of user uid. It outputs a secret key SK\(_{uid,auid}\) for the user uid.

**Phase 3: Data Encryption**

- **Encrypt**\((SP, \{PK\}_{k\in I_A}, \{PK_{aid}\}_{x\in S_A}, m, A)\) \(\rightarrow\) CT. The encryption algorithm takes as inputs the system parameter SP, a set of public keys \(\{PK_{k}\}_{k\in I_A}\) from the involved authority set \(I_A\), a set of public attribute keys \(\{PK_{aid}\}_{x\in S_A}\), the data \(m\) and an access structure \(A\) over all the selected attributes from the involved AAs. The algorithm first encrypts the data \(m\) by using symmetric encryption methods with a content key \(k\). Then, it encrypts the content key \(k\) under the access structure \(A\) and outputs a ciphertext CT. We will assume that the ciphertext implicitly contains the access structure \(A\).

**Phase 4: Data Decryption**

The data decryption phase consists of Decryption Token Generation by cloud servers and Data Decryption by users with the following algorithms:

- **TKGen**(CT, GPK\(_{uid}\), \{SK\(_{uid,k}\)\}_{k\in I_A}) \(\rightarrow\) TK. The decryption token generation algorithm takes as inputs the ciphertext CT which contains an access structure \(A\), user’s global public key GPK\(_{uid}\) and a set of user’s secret keys \(\{SK_{uid,k}\}_{k\in I_A}\). If the user uid holds sufficient attributes that satisfy the access structure \(A\), the algorithm can successfully compute the correct decryption token TK for the ciphertext CT.
- **Decrypt**(CT, TK, GSK\(_{uid}\)) \(\rightarrow\) m. The decryption algorithm takes as inputs the ciphertext CT, the decryption token TK and the user’s global secret key GSK\(_{uid}\). It first decrypts the content key and further uses the content key to decrypt the data. It outputs the data m.

**Phase 5: Attribute Revocation**

This phase contains three steps: Update Key Generation by AAs, Secret Key Update by Non-revoked Users\(^1\) and Ciphertext Update by Servers.

- **UKeyGen**(SK\(_{uid}\), \{u\}_j, \{VK\}_{x\_aid}) \(\rightarrow\) \{KUK\_j\_aid, CUK\_j\_aid\}. The update key generation algorithm takes as inputs the secret authority key SK\(_{aid}\), a set of user’s secret \(\{u\}_j\) and the previous version key of the revoked attribute VK\(_{x\_aid}\). It outputs both the user’s Key Update Key KUK\(_{j\_aid}\) \((j \in S_U, j \neq \mu, x_{\_aid} \in S_{j\_aid})\) and the Ciphertext Update Key CUK\(_{j\_aid}\).
- **SKUpdate**(SK\(_{uid,auid}, KUK\(_{uid,auid}\)) \(\rightarrow\) SK’\(_{uid,auid}\). The user’s secret key update algorithm takes as inputs the current secret key SK\(_{uid,auid}\) and its key update key KUK\(_{uid,auid}\). It outputs a new secret key SK’\(_{uid,auid}\).
- **CTUpdate**(CT, CUK\(_{x\_aid}\)) \(\rightarrow\) CT’. The ciphertext update algorithm takes as inputs the current ciphertext CT and the ciphertext update key CUK\(_{x\_aid}\). It outputs a new ciphertext CT’.

**C. Security Assumption of Each Entity**

In DAC-MACS, we have the following assumptions:

- The CA is trusted, but it is not allowed to decrypt any ciphertexts.
- Each AA is also trusted, but it can be corrupted by the adversary.
- The server is semi-trusted (curious but honest). It will not deny service to any authorized users, and will correctly execute the tasks assigned by the AA. But it is curious about the data content or the received messages.
- Users are dishonest and may collude to obtain unauthorized access to data.
- All the non-revoked users will not give the received update keys to the revoked user\(^2\).

**D. Decisional q-parallel Bilinear Diffie-Hellman Exponent Assumption**

We recall the definition of the decisional q-parallel Bilinear Diffie-Hellman Exponent problem in \([5]\) as follows. Let \(a, b_1, \ldots, b_q \in \mathbb{Z}_p\) be chosen at random and \(g\) be a generator of \(\mathbb{G}\). If an adversary is given

\[
y = (g^a g^{b_1}, g^{a^2}, g^{a^{b_2}}, \ldots, g^{a^{b_q}}, g^{a^{b_1} a^{b_2}} \cdots g^{a^{b_q}})\]

\[\forall 1 \leq j \leq q\; g^{a^{b_j}} b_j, g^{a^2} b_j, \ldots, g^{a^{b_q} b_j} b_j, \ldots, g^{a^{b_q}} b_j)\]

\[\forall 1 \leq j,k \leq q, j \neq k\; g^{a^{b_j} b_k}, \ldots, g^{a^{b_q} b_k} b_j),\]

\(^1\)Users who hold the revoked attribute but have not been revoked.

\(^2\)We will remove this assumption in the extensive data access control scheme (EDAC-MACS) in Section V.
it must be hard to distinguish a valid tuple $e(g, g)^{\rho_1 \ast} \in G_T$
from a random element $R$ in $G_T$.

An algorithm $B$ that outputs $z \in \{0, 1\}$ has advantage $\varepsilon$ in solving q-parallel BDHE in $G$ if

$$\left| \Pr[B(\tilde{y}, T = e(g, g)^{\rho_1 \ast}) = 0] - \Pr[B(\tilde{y}, T = R) = 0] \right| \geq \varepsilon.$$ 

**Definition 2.** The decisional q-parallel BDHE assumption holds if no polynomial time algorithm has a non-negligible advantage in solving the q-parallel BDHE problem.

### E. Security Model

We now describe the security model of DAC-MACS by the following game between a challenger and an adversary. The security model allows the adversary to query for any secret keys and update keys that cannot be used to decrypt the challenge ciphertext. Similar to [10], we assume that the adversary can corrupt authorities only statically, but key queries are made adaptively. Let $S_A$ denote the set of all the authorities. The security game is defined as follows.

**Setup.** The system parameters are generated by running the CA setup algorithm. The adversary specifies a set of corrupted attribute authorities $S_A' \subset S_A$. The challenger generates the public keys by querying the AA setup oracle, and generates the secret keys by querying the secret key generation oracle. For uncorrupted authorities in $S_A - S_A'$, the challenger sends only the public keys to the adversary. For corrupted authorities in $S_A'$, the challenger sends both public keys and secret keys to the adversary.

**Phase 1.** The adversary makes secret key queries by submitting pairs $(uid, S_{uid})$ to the challenger, where $S_{uid} = \{S_{uid,k} \mid k \in S_A - S_A'$ is a set of attributes belonging to several uncorrupted AAs. The challenger gives the corresponding secret keys $\{S_{K,uid,k} \}$ to the adversary. The adversary also makes update key queries by submitting a set of attributes $S_{aid}$. The challenger gives the corresponding update keys to the adversary.

**Challenge.** The adversary submits two equal length messages $m_0$ and $m_1$. In addition, the adversary gives a challenge access structure $(M^*, \rho^*)$ which must satisfy the following constraints. We let $V$ denote the subset of rows of $M^*$ labeled by attributes controlled by corrupted AAs. For each $uid$, we let $V_{uid}$ denote the subset of rows of $M^*$ labeled by attributes that the adversary has queried. For each $uid$, we require that the subspace spanned by $V \cup V_{uid}$ must not include $(1, 0, \ldots, 0)$. In other words, the adversary cannot ask for a set of keys that allow decryption, in combination with any keys that can be obtained from corrupted AAs. The challenger then flips a random coin $b$, and encrypts $m_b$ under the access structure $(M^*, \rho^*)$. Then, the ciphertext $CT^*$ is given to the adversary.

**Phase 2.** The adversary may query more secret keys and update keys, as long as they do not violate the constraints on the challenge access structure $(M^*, \rho^*)$ and the following constraints: None of the updated secret keys (generated by the queried update keys and the queried secret keys) is able

3There is another reason that makes the queried secret keys cannot decrypt the challenge ciphertext. That is at least one of the attributes in the previous queried secret keys may be not in the current version.

to decrypt the challenged ciphertexts. In other words, the adversary is not able to query the update keys that can update the queried secret keys to the new secret keys that can decrypt the challenge ciphertext.

**Guess.** The adversary outputs a guess $b'$ of $b$.

The advantage of an adversary $A$ in this game is defined as $Pr[b' = b] - \frac{1}{2}$.

**Definition 3.** DAC-MACS is secure against static corruption of authorities if all polynomial time adversaries have at most a negligible advantage in the above security game.

**Definition 4.** DAC-MACS is collusion resilience if no polynomial time adversaries can decrypt the data by combining attributes of different users together, when each individual user cannot decrypt the data only with its own attributes.

### III. DAC-MACS: Data Access Control for Multi-Authority Cloud Storage

This section first gives an overview of our scheme. Then, we describe DAC-MACS which consists of five phases.

#### A. Overview

Although the existing multi-authority CP-ABE scheme [10] proposed by Lewko and Waters has high policy expressiveness and has been extended to support attribute revocation in [12], it still cannot be applied to access control for multi-authority cloud storage systems due to the inefficiency of decryption and revocation. Thus, the main challenge is to construct a new underlying multi-authority CP-ABE scheme that supports efficient decryption and revocation.

To design a multi-authority CP-ABE scheme, the most challenging issue is how to tie different secret keys together but still prevent the collusion attack. Similar to [7], in DAC-MACS, we separate the authority into a global certificate authority (CA) and multiple attribute authorities (AAs). The CA sets up the system and assigns a global user identity $uid$ to each user and a global authority identity $aid$ to each attribute authority. The global unique $uid$ can tie secret keys issued by different AAs together for decryption, and the global unique $aid$ can distinguish attributes issued by different AAs. Thus, by using $uid$ and $aid$, the collusion attack can be resisted. However, different from [7], the CA in DAC-MACS is not involved in any attribute management and the creation of secret keys reflecting the user’s attributes. DAC-MACS also requires all the AAs to generate their own public keys which can be used to encrypt data together with the global public parameters, instead of only using the system unique public key for data encryption. This solves the security drawback in [7], i.e., it prevents the CA from decrypting the ciphertexts.

To achieve efficient decryption on the user, we propose a token-based decryption outsourcing method. We apply the decryption outsourcing idea from [12] and extend it to multiple authority systems by letting the CA generate a pair of global secret key and global public key for each legal user in the system. During the decryption, the user submits its secret keys issued by AAs to the server and asks the server to compute a decryption token for the ciphertext. The user can decrypt
the ciphertext by using the decryption token together with its global secret key.

To solve the attribute revocation problem, we assign a version number for each attribute, such that for each attribute revocation, only those components associated with the revoked attribute in secret keys and ciphertexts need to be updated. When an attribute is revoked from a user, the corresponding AA will generate a new version key for this revoked attribute, and computes an update key containing a Ciphertext Update Key (CUK) and several user’s Key Update Keys (KUKs). With the KUKs, each non-revoked user can update its secret key to the current version, while the revoked user cannot update its secret key even using other users’ update keys, since each KUK is associated with the **uid** (Backward Security). The ciphertexts can also be updated to the current version with the **uid** (Forward Security). Moreover, all the users only need to hold the latest secret key, rather than all the previous secret keys. To improve the efficiency, we delegate the workload of ciphertext update to the server by using the proxy re-encryption method.

**B. System Initialization**

This phase consists two steps: CA Setup and AA Setup.

1) **CA Setup:** Let \( S_A \) and \( S_U \) denote the set of attribute authorities and the set of users in the system respectively. Let \( G \) and \( G_T \) be the multiplicative groups with the same prime order \( p \) and \( e: G \times G \rightarrow G_T \) be the bilinear map. Let \( g \) be the generator of \( G \). Let \( H: \{0, 1\}^* \rightarrow G \) be a hash function such that the security is in the random oracle.

The CA runs the CA setup algorithm, which takes a security parameter as input. The CA first generates a pair of signature and verification key \((sk_{CA}, vk_{CA})\). Then, it chooses a random number \( a \in Z_p \) as the master key MSK of the system and compute the system parameter as

\[
SP = (g, g^a, G, G_T, H).
\]

The CA accepts both User Registration and AA Registration:

- **User Registration**

  Every user should register itself to the CA during the system initialization. The CA runs the user registration algorithm UserReg which takes the system parameter and the user information as inputs. If the user is legal in the system, it assigns a global user identity **uid** to this user, and generates the global public key \(GPK_{uid} = g^{auid} \) and the global secret key \( GSK_{uid} = z_{uid} \) by randomly choosing two numbers \( u_{uid}, z_{uid} \in Z_p \). The CA also generates a certificate \( Cert(uid) \) which contains an item \( Sign_{sk_{CA}}(uid, u_{uid}, z^{1/|\mu|}) \). Then, the CA sends the global public/secret key pair \((GPK_{uid}, GSK_{uid})\) and the certificate \( Cert(uid)\) to user **uid**.

- **AA Registration**

  Each AA should also register itself to the CA during the system initialization. The CA runs the AA registration algorithm AAReg by taking the information of AA as input. If the AA is a legal authority in the system, the CA first assigns a global authority identity **aid** to it. Then, the CA sends both its verification key \( vk_{CA} \) and the system parameter \( SP \) to this AA.

2) **AA Setup:** Each \( AA_k (k \in S_A) \) runs the AA setup algorithm AASetup. Let \( S_A \) denote the set of all attributes managed by this authority \( AA_k \). It chooses three random numbers \( \alpha_k, \beta_k, \gamma_k \in Z_p \) as the secret authority key \( SK_k = (\alpha_k, \beta_k, \gamma_k) \). For each attribute \( x_i \in S_A \), the authority generates a public attribute key as

\[
PK_{x_i} = (g^{\alpha_k} H(x_i))^{\gamma_k}
\]

by implicitly choosing an attribute version key as \( VK_{x_i} = v_{x_i} \).

The \( AA_k \) also computes the public authority key as

\[
PK_k = \left(e(g, g)^{\alpha_k}, g^{\frac{1}{\gamma_k}}, g^{\gamma_k}\right).
\]

All the public attribute keys and public authority keys are published on the public bulletin board of \( AA_k \).

**C. Secret Key Generation by AAs**

For every user \( U_j (j \in S_U) \), each \( AA_k (k \in S_A) \) first authenticates whether this user is a legal user by verifying its certificate by using the verification key \( vk_{CA} \). If the user is not legal, it aborts. Otherwise, the \( AA_k \) assigns a set of attributes \( S_{j,k} \) to this user according to its role or identity in its administration domain. Then, the \( AA_k \) runs the secret key generation algorithm SKKeyGen to generate the user’s secret key \( SK_{j,k} \).

The algorithm takes as inputs the secret authority key \( SK_{uid} \), the system parameter \( SP \), the set of public attribute keys \( \{PK_{uid}\} \), a set of attributes \( S_{uid,aid} \) that describes the secret key, and the certificate of user **uid**. It chooses a random number \( t_{j,k} \in Z_p \) and computes \( SK_{j,k} \) as

\[
SK_{j,k} = (K_{j,k} = g^{a_{uid}}, g^{a_{uid} t_{j,k}}, L_{j,k} = g^{\frac{a_{uid}}{t_{j,k}}}, R_{j,k} = g^{a_{uid}}, \forall x_{j,k} \in S_{j,k} : K_{j,x_{j,k}} = g^{\frac{a_{uid}}{t_{j,k}}}, (g^{\alpha_k} \cdot H(x_{j,k}))^{\gamma_k r_{uid}}, \text{ where } j \in S_U \text{ and } k \in S_A.
\]

**D. Data Encryption by Owners**

Before outsourcing data \( m \) into the cloud, the owner encrypts the data by running the data encryption algorithm Encrypt. It takes as inputs the system parameter \( SP \), a set of public keys \( \{PK_k\}_{k \in S_A} \) from the involved authority set \( I_A \), a set of public attribute keys \( \{PK_{uid} \}_{uid \in S_{uid}} \), the data \( m \) and an access structure \((M, \rho)\) over all the selected attributes from the involved AAs. Let \( M \) be a \( l \times n \) matrix, where \( l \) denotes the total number of all the attributes. The function \( \rho \) associates rows of \( M \) to attributes.

The algorithm first divides the data into several data components as \( m = \{m_1, \cdots, m_m\} \) according to the logic granularities. For example, the personal data may be divided into \{name, address, security number, employer, salary\}. It then encrypts data components with different symmetric content keys \( \{k_1, \cdots, k_m\} \) by using symmetric encryption methods, where \( k_i \) is used to encrypt \( m_i (i = 1, \cdots, n) \).

Then, it defines an access structure \( M \) and encrypts each content key \( k_i (i = 1, \cdots, n) \) under this access structure. For
computes the ciphertext as
\[ CT = (E_n(m), C = \kappa \cdot \prod_{k \in I_A} (g(g, g^{\alpha_k})^{s_k})^{s_k}, C' = g^{s}, C'' = g^{\kappa C}, \]
\[ \forall i = 1 \rightarrow l : C_i = g^{\alpha_i}, ((g^{\rho(i)} H(\rho(i))^2)^{r_i}, D_{1, i} = g^{\kappa s_k}, D_{2, i} = g^{\kappa r_i}, \rho(i) \in S_{A_k}). \]
In real systems, if the data \( m \) is divided into \( n \) components, the ciphertext \( CT \) also consists of \( n \) components \( CT = \{CT_1, \ldots, CT_n\}. \)

**E. Data Decryption by Users (with the help of Cloud)**

All the legal users in the system can freely query any interested ciphertexts from the cloud server. But only when the user’s attributes satisfy the access structure embedded in the ciphertext, he/she is able to decrypt the content keys and further use them to decrypt the data. This phase consists of two steps: *Token Generation by Cloud Server* and *Data Decryption by Users*.

1) **Token Generation by Cloud Server:** The user \( U_j (j \in S_U) \) sends its secret keys \( \{SK_{j,k} \}_{k \in S_{A_k}} \) to the server and asks for a decryption token for the ciphertext \( CT \). Only when the attributes the user \( U_j \) possesses satisfy the access structure defined in the ciphertext \( CT \), the server can successfully compute the correct decryption token \( TK \).

The server runs the token generation algorithm TKGen, which takes as inputs the ciphertext CT (which contains an access structure \( A \), user’s global public key \( GPK_j \) and a set of user’s secret keys \( \{SK_{j,k} \}_{k \in S_{A_k}} \). Let \( I = \{I_{A_k} \}_{k \in S_{A_k}} \) be the whole index set of all the attributes involved in the ciphertext, where \( I_{A_k} \subseteq \{1, \ldots, l\} \) is the index subset of the attributes from the \( A_{A_k} \), defined as \( I_{A_k} = \{i : \rho(i) \in S_{A_k} \}. \) Let \( N_{A_k} = |I_k| \) be the number of AAs involved in the ciphertext. It chooses a set of constants \( \{w_i \in \mathbb{Z}_p\}_{i \in I} \) and reconstructs the encryption exponent as \( s = \sum_{i \in I} w_i I_{A_k} \) if \( \{I_{A_k} \} \) are valid shares of the secret \( s \) according to \( M \).

The algorithm computes the decryption token TK as
\[ TK = \prod_{k \in I_{A_k}} \prod_{i \in I_{A_k}} \frac{e(C_i, GPK_j) \cdot e(D_{1, i, K_{j, \rho(i)}}) \cdot e(D_{2, i, L_{j, i}}))}{\mu N_{A_k} \cdot e(g, g)^{\alpha_k s}}. \]
\[ = \prod_{k \in I_{A_k}} e(g, g)^{\frac{\alpha_k s}{w_i N_{A_k}}} \cdot e(g, g)^{\frac{\alpha_k}{w_i}}. \]

It outputs the decryption token TK for the ciphertext CT and sends it to the user \( U_j \).

2) **Data Decryption by Users:** Upon receiving this decryption token TK, the user \( U_j \) can use it to decrypt the ciphertext together with its global secret key \( GSK_j = z_j \) as
\[ \kappa = C / TK^{\gamma_i}. \]
Then, the user can use the content key \( \kappa \) to further decrypt the data as
\[ m = Dec_{\kappa}(Enc_{\kappa}(m)). \]

**F. Efficient Attribute Revocation**

Suppose an attribute \( \tilde{\kappa}_j \) of the user \( U_\mu \) is revoked from the \( A_{A_k} \). The attribute revocation includes three phases: *Update Key Generation by AAs, Secret Key Update by Non-revoked Users* and *Ciphertext Update by Cloud Server*. The secret key update can prevent the revoked user from decrypting the new ciphertexts which are encrypted by the new public attribute keys (Backward Security). The ciphertext update can also guarantee that the newly joined user who have sufficient attributes can still access the previous published data (Forward Security).

1) **Update Key Generation by AAs:** The corresponding authority \( A_{A_k} \) runs the update key generation algorithm UKeyGen to compute the update keys. The algorithm takes as inputs the secret authority key \( SK_k \), the current attribute version key \( V_{\kappa} \) and the user’s global public keys \( GPK_j \). It generates a new attribute version key \( V_{\kappa} \) and then applies it to compute the user’s Update Key Key as
\[ KUK_{j, \tilde{\kappa}} = g^{\mu \beta_k \cdot AUK_{\tilde{\kappa}}}. \]
and the Ciphertext Update Key as
\[ \begin{align*} 
\text{CUK}_{j, \tilde{\kappa}} &= g^{\mu \beta_k \cdot AUK_{\tilde{\kappa}} / \gamma_k}. 
\end{align*} \]

2) **Secret Key Update by Non-revoked Users:** For each non-revoked user \( U_j (j \in S_U, j \neq \mu) \) who holds the revoked attribute \( \tilde{\kappa}_j \), the \( A_{A_k} \) sends the corresponding user’s key update key \( KUK_{j, \tilde{\kappa}} \) to it. Upon receiving \( KUK_{j, \tilde{\kappa}} \), the user \( U_j \) runs the key update algorithm SKUpdate to update its secret key as
\[ \begin{align*} 
\text{SK}_{j, \tilde{\kappa}} &= (K'_{j, k} = K_{j, k}, L'_{j, k} = L_{j, k}, R_{j, k} = R_{j, k}, K'_{j, \tilde{\kappa}} = K_{j, \tilde{\kappa}}, \forall x \in S_{A_k}, x \neq \tilde{\kappa} : K'_{j, k} = K_{j, k}). 
\end{align*} \]

3) **Ciphertext Update by Cloud Server:** The \( A_{A_k} \) sends a ciphertext update key \( CUK_{\tilde{\kappa}} \) to the server. Upon receiving the \( CUK_{\tilde{\kappa}} \), the server runs the ciphertext update algorithm CTUpdate to update all the ciphertexts which are associated with the revoked attribute \( \tilde{\kappa}_j \). It takes inputs as the current ciphertext CT and the CTUpdate. It only needs to update several components of the ciphertext, which are associated with the
\[\tilde{x}_k \text{. The new ciphertext } C T' \text{ is published as} \]
\[C T' = (E n_k(m), C = \kappa \cdot \prod_{k \in K} e(g, g)^{a_k})^i, \quad C' = g^\gamma, \quad C'' = g^{\frac{\gamma}{2}} , \quad \forall i = 1 \text{ to } l : \]
\[\text{if } p(i) \neq \tilde{x}_k : \quad C_i = g^{\epsilon_{\text{香菇}}} \cdot ((g^{\epsilon_{\text{香菇}}} H(x_i))^{\frac{\gamma}{2}})^{-r_i}, \]
\[D_{1,i} = g^{\epsilon_{\text{香菇}}}, \quad D_{2,i} = g^{\frac{\gamma}{2}} r_i, \]
\[\text{if } p(i) = \tilde{x}_k : \quad C'_i = C_i \cdot D_{1,i}^{\text{Cuk}} \cdot \tilde{\gamma}, \]
\[D_{1,i} = g^{\frac{\gamma}{2}}, \quad D_{2,i} = g^{\frac{\gamma}{2}} r_i . \]

DAC-MACS requires to update only a few components which are associated with the revoked attribute, while the other components are not changed. This can greatly improve the efficiency of attribute revocation.

The ciphertext update not only can guarantee the forward security of the attribute revocation, but also can reduce the storage overhead on users (i.e., all the users only need to hold the latest secret key, rather than to keep records on all the previous secret keys).

IV. ANALYSIS OF DAC-MACS

This section provides a comprehensive analysis of DAC-MACS, followed by security and performance analysis.

A. Comprehensive Analysis

Let |p| be the size of element in the groups with the prime order p. Let t_1 be the total number of attributes in a ciphertext and t_2 be the total number of attributes of a user. Let n_u denote the number of users in the system. For the revoked attribute x, let n_{non,x} be the number of non-revoked users who hold the revoked attribute and let n_{c,x} be the number of ciphertexts which contain the revoked attribute.

Table I shows the comparison among our DAC-MACS and two existing schemes, all of which relied on the ciphertext re-encryption to achieve the attribute revocation. From the table, we can see that DAC-MACS incurs less computation cost for the decryption on the user and less communication cost for the revocation. In DAC-MACS, the attribute revocation is controlled and enforced by each AA independently, but the ciphertexts are updated by the semi-trusted server, which can greatly reduce the workload on owners. For the security of attribute revocation, DAC-MACS can achieve both forward security and backward security. The cloud server in our system is required to be semi-trusted. Even if the cloud server is not semi-trusted in some scenarios, the server will not update the ciphertexts correctly. In this situation, the forward security cannot be guaranteed, but our system can still achieve the backward security, i.e., the revoked user cannot decrypt new ciphertexts that requires the revoked attributes for decryption.

\[\begin{align*}
\text{TABLE I} & \quad \text{COMPREHENSIVE COMPARISON OF CP-ABE WITH ATTRIBUTE REVOCATION SCHEMES} \\
\text{Scheme} & \quad \text{Authority} & \quad \text{Computation} & \quad \text{Revocation} & \quad \text{Revocation Security} & \quad \text{Revocation Controller} & \quad \text{Ciphertext} \\
& & \quad \text{Encrypt} & \quad \text{Decrypt} & \quad \text{Message (|p|)} & \quad \text{Backward} & \quad \text{Forward} & \quad \text{Updater} \\
& \quad \text{Controller} & \quad \text{Owner} & \quad \text{Server}^* & \quad \text{Server}^† & \quad \text{Owner} & \quad \text{Owner} & \quad \text{Owner} & \quad \text{Server}^\
\text{Hur’s [13]} & \quad \text{Single} & \quad O(t_1 + \log n_u) & \quad O(t_2) & \quad O(n_{non,x} \log \frac{t_2}{n_{non,x}}) & \quad \text{Yes} & \quad \text{Yes} & \quad \text{Server}^* & \quad \text{Server}^† \\
\text{DACC [14]} & \quad \text{Multiple} & \quad O(t_1) & \quad O(t_2) & \quad O(n_{c,x} \cdot n_{non,x}) & \quad \text{Yes} & \quad \text{No} & \quad \text{Owner} & \quad \text{Owner} \\
\text{DAC-MACS} & \quad \text{Multiple} & \quad O(t_1) & \quad O(t_2) & \quad O(1) & \quad O(n_{non,x}) & \quad \text{Yes} & \quad \text{Yes} & \quad \text{AA} & \quad \text{Server}^† \\
\end{align*}\]

*: The decryption computation on the user; †: The server is fully trusted; ‡: The server is semi-trusted.

B. Security Analysis

Under the security model we defined in Section II, we prove that DAC-MACS is provable secure and collusion resilience, as concluded in the following theorems:

**Theorem 1.** When the decisional q-parallel BDHE assumption holds, no polynomial time adversary can selectively break DAC-MACS with a challenge matrix of size l^* × n^*, where n^* < q.

**Proof:** Suppose we have an adversary A with non-negligible advantage \( \varepsilon = \text{Adv}_A \) in the selective security game against our construction and suppose it chooses a challenge matrix \( M^* \) with the dimension at most \( q - 1 \) columns. In the security game, the adversary can query any secret keys and update keys that cannot be used for decryption in combination with any keys it can obtain from the corrupted AAs. With these constraints, the security game in multi-authority systems can be treated equally to the one in single authority systems. Similarly, we can build a simulator B that plays the decisional q-parallel BDHE problem with non-negligible advantage. The detailed proof is described in the full version of this paper [11].

**Theorem 2.** DAC-MACS is secure against the collusion attack of users.

**Proof:** In DAC-MACS, each user in the system is assigned with a global unique identity uid, and all the secret keys issued to the same user from different AAs are associated with the uid of this user. Thus, it is impossible for two or more users to collude and decrypt the ciphertext. Moreover, due to the unique uid of each AA, all the attributes are distinguishable, even though some AAs may issue the same attribute. This can prevent the user from replacing the components of a secret key issued by an AA with those components from other secret keys issued by another AA.

**Privacy-Preserving Guarantee** Due to the decryption outsourcing, the server can get the users’ secret keys. However, the server still cannot decrypt the ciphertext without the knowledge of the users’ global secret keys. Moreover, the ciphertext update is done by using the proxy re-encryption method, thus the server does not need to decrypt the ciphertext.
The communication cost of the owner versus the number of attributes from each AA is set to be 10. Fig. 2(b) gives the comparison of encryption time on the owner versus the number of attributes from each AA, where the involved number of attributes from each AA is set to be 10. Suppose the user has the same number of attributes in each AA. Fig. 2(c) shows the comparison of decryption time on the user versus the number of attributes, where the number of attributes is 10. Fig. 2(d) describes the computation time on the user versus the number of attributes the user holds from each AA, where the number of attributes is 10. Fig. 2(e) gives the comparison of ciphertext re-encryption/update versus the number of revoked attributes appeared in the ciphertext. The simulation results show that our DAC-MACS incurs less computation cost on the encryption of owners, the decryption of users and the re-encryption of ciphertexts.

V. Extensive DAC-MACS

In DAC-MACS, there is a security assumption that all the non-revoked users will not give the received update keys to the revoked user. However, this is a strong assumption, and in practical the revoked user may collude with other users to obtain the update keys. This section first proposes an extensive data access control scheme (EDAC-MACS), and then give the security analysis to show that EDAC-MACS can achieve the same security goal without this assumption.

A. EDAC-MACS

If we remove this assumption, the backward security in DAC-MACS will no longer be guaranteed. That is when the adversary \( \mu \) (the revoked user) corrupted any AA, he/she could...
obtain all the users’ secrets \( \{ u_{uid} \} \), and use it to transfer the other user’s key update key \( \text{KUK}_{j,k} \) to its own one as 

\[
\text{KUK}_{A,j,k} = (\text{KUK}_{j,k})^{u_{\text{uid}}}_A.
\]

Then, the adversary can use it to update his secret key to the latest version by running the secret key update algorithm \( \text{SKUpdate} \).

To address this security issue, we modify the secret key generation algorithm \( \text{SKGen} \) by padding a new piece to the \( K_{j,k} \). It generates the user’s secret key as 

\[
\text{SK}_{j,k} = (K_{j,k} = g^{\frac{a_k}{j}} \cdot g^{a_{\text{uid}} j} \cdot g^{\frac{b_k}{j}})^k, \quad \forall x \in S_j : L_{j,k,x} = g^{\frac{b_k}{j}(j+s+v_k)},
\]

\[
K_{j,k} = g^{\frac{b_k}{j}(j+s+v_k)} \cdot (g^{v_k} \cdot H(x_k))^{\frac{1}{j}}b_k u_j,
\]

where \( j \in S_U \) and \( k \in S_A \).

The encryption algorithm Encrypt is the same as DAC-MACS, but during the data decryption, the decryption token \( TK \) is generated by the new decryption token generation algorithm \( \text{TKGen} \) as 

\[
\text{TK} = \prod_{k \in A} e(C', K_{j,k}) \cdot e(R_{j,k}, C')^{-1} = \prod_{k \in A} e(g,g)^{a_{\text{uid}} N_A} \cdot \prod_{k \in A} e(g,g)^{\frac{a_k}{s}}.
\]

Correctness We observe that 

\[
\prod_{k \in A} e(C', K_{j,k})e(R_{j,k}^{-1}, C') = \prod_{k \in A} e(g,g)^{a_{\text{uid}} N_A} \cdot \prod_{k \in A} e(g,g)^{\frac{a_k}{s}}.
\]

For each \( i \in I \), suppose \( \rho(i) \in S_k \), it computes 

\[
e(C_i, \text{GPK}_k) \cdot e(D_{1,i}, K_{j,k,\rho(i)}) \cdot e(D_{2,i}, L_{j,k,\rho(i)})
\]

\[
= e(g,g)^{a_{\text{uid}} N_A} \cdot \prod_{k \in A} e(g,g)^{\frac{a_k}{s}}.
\]

Then, it computes 

\[
\prod_{k \in A} e(g,g)^{a_{\text{uid}} N_A} = e(g,g)^{a_{\text{uid}} N_A}.
\]

The decryption algorithm is the same as DAC-MACS.

During the attribute revocation, the authority also needs to first generate the update keys. The ciphertext update key \( \text{CUK}_{i,k} \) is the same as the one in DAC-MACS. However, the user’s key update key is generated as 

\[
\text{KUK}_{j,k} = (\text{KUK}_{1,j,k} = g^{(a_k+b_j)} \cdot \text{AUK}_{i,k},
\]

\[
\text{KUK}_{2,j,k} = g^{(a_k+b_j)} \cdot \text{AUK}_{i,k}).
\]

Accordingly, the key update algorithm \( \text{SKUpdate} \) is constructed as 

\[
\text{SK}'_{j,k} = (K'_{j,k} = K_{j,k}, R'_{j,k} = R_{j,k},
\]

\[
L'_{j,k,k} = L_{j,k,k} \cdot \text{KUK}_{j,k} \cdot \text{KUK}_{1,j,k},
\]

\[
\forall x \in S_A, x \neq \tilde{x} : L'_{j,k,x} = L_{j,k,x}, K'_{j,k} = K_{j,k}).
\]

B. Security Analysis

We conclude the security of EDAC-MACS as the following two theorems:

**Theorem 3.** In EDAC-MACS, the revoked user has no chance to update its secret key, even if it can corrupt some AAs (not the AA corresponding to the revoked attribute) and collude some non-revoked users.

**Proof:** In EDAC-MACS, each key update key is associated with the user’s identity \( \text{uid} \). And the item \( g^{b_k \cdot v_k / j} \) in the secret key prevents users from updating their secret keys with the other user’s update key, since \( v_k \) is only known by the AA \( k \) and kept secret to all the users. ■

**Theorem 4.** When the decisional \( q \)-parallel BDHE assumption holds, no polynomial time adversary can selectively break EDAC-MACS with a challenge matrix of size \( l^* \times n^* \), where \( n^* < q \).

**Proof:** Based on the Theorem 3, the security proof of EDAC-MACS is similar to DAC-MACS as Theorem 1. The main difference is how to simulate the new secret keys in EDAC-MACS. The detailed security proof is described in Appendix A. ■

VI. RELATED WORK

Cryptographic techniques are well applied to access control for remote storage systems [15–17]. To prevent the untrusted servers from accessing sensitive data, traditional methods [18], [19] usually encrypt the data and only the users who hold valid keys can decrypt. Then, the data access control becomes the matter of key distribution. Although these methods can
provide secure data access control, the key management is very complicated when more users are in the system. Data owners also have to stay online all the time to deliver keys to new users. Moreover, for each data, there are multiple copies of ciphertexts for users with different keys, which will incur high storage overhead on the server. Some methods [20] deliver the key management and distribution from data owners to the remote server under the assumption that the server is trusted. However, the server is not fully trusted in cloud storage systems and thus these methods cannot be applied to data access control for cloud storage systems.

Attribute-based Encryption (ABE) is a promising technique that is designed for access control of encrypted data. After Sahai and Waters introduced the first ABE scheme [21], Goyal et al. [22] formulated the ABE into two complimentary forms: Key-Policy ABE (KP-ABE) and Ciphertext-Policy ABE (CP-ABE). There are a number of works used ABE to realize fine-grained access control for outsourced data [13], [23], [24]. These schemes require a trusted authority to manage all the attributes in the system and issue secret keys to users. Since the authority can decrypt all the encrypted data, it becomes a vulnerable security point for the system. Moreover, the authority may become the performance bottleneck in the large scale cloud storage systems.

Some cryptographic methods are proposed for the multi-authority ABE problem [7]–[10], [25], [26], where there are multiple authorities coexist and users may have attributes from multiple authorities. However, some of them [7], [8] still require a global authority. Lin et al. [25] proposed a decentralized scheme based on threshold mechanism. In this scheme, the set of authorities is pre-determined and it requires the interaction among the authorities during the system setup. In [10], Lewko et al. proposed a new comprehensive scheme, which does not require any central authority. However, they did not consider attribute revocation problem.

There are a number of works about the revocation in ABE systems in the cryptography literature [2]–[6]. However, these methods either only support the user level revocation or rely on the server to conduct the attribute revocation. Moreover, these attribute revocation methods are designed only for ABE systems with single authority. Ruj et al. [14] designed a DACC scheme and proposed an attribute revocation method for the Lewko and Waters’ decentralized ABE scheme. However, their attribute revocation method incurs a heavy communication cost since it requires the data owner to transmit a new ciphertext component to every non-revoked user. Li et al. [27] proposed an attribute revocation method for multi-authority ABE systems, but their methods is only for KP-ABE systems.

Green et al. [12] proposed two ABE schemes that outsource the decryption to the server. In their schemes, the authority separate the traditional secret key into a user secret key and a transformation key. However, their schemes are designed only for the single authority systems and do not support for the multi-authority systems. That is because each authority may generate different user’s secret key, such that the transformation keys cannot be combined together to transform the ciphertext into a correct intermediate value.

VII. Conclusion

In this paper, we proposed an effective data access control scheme for multi-authority cloud storage systems, DAC-MACS. We also constructed a new multi-authority CP-ABE scheme, in which the main computation of decryption is outsourced to the server. We further designed an efficient attribute revocation method that can achieve both forward security and backward security.

We also removed the security assumption that all the non-revoked users will not reveal their received key update keys to the revoked user. We further proposed an extensive DAC-MACS, which is secure under weaker security assumptions. Although this work is for multi-authority cloud storage systems, the techniques designed in this paper can be applied into other applications, such as any remote storage systems, online social networks etc.

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Therefore, we can build a simulator against our construction and suppose it chooses a challenge matrix $\mathbf{r}$ and gives the algorithm the challenge access control in multi-authority systems in $\text{ASIACCS}'10$. ACM, 2010, pp. 261–270.

The simulator assigns a user identity $uid$ to the adversary and chooses two random numbers $u'_{uid}, z_{uid} \in \mathbb{Z}_p$. Then, it sets $GSK_{uid} = z_{uid}$ and implicitly sets $u_{uid} = u'_{uid} - a/r_{uid}$ by setting

$$GPK_{uid} = \frac{g^{u'_{uid}} (g^{a})^{\frac{1}{r_{uid}}}}{a_{uid}}.$$

The simulator then sends the global public/secret key pairs $(GPK_{uid}, GSK_{uid})$ to the adversary.

**Phase 1.** In this phase, the simulator answers secret key queries and update key queries from the adversary. Suppose the adversary makes secret key queries by submitting pairs $(uid, S_k)$ to the simulator, where $S_k$ is a set of attributes belonging to an uncorrupted authority $AA_k$. Suppose $S_k$ does not satisfy $M^*$ together with any keys that can obtain from corrupted authorities.

The simulator finds a vector $\vec{w} = (w_1, w_2, \ldots, w_{mp}) \in \mathbb{Z}_p^n$, such that $w_1 = -1$ and for all $i$ where $p^*(i) \in S_k$ we have that $\vec{w} \cdot M'_i = 0$. By the definition of a LSSS, such a vector must exist, since $S_k$ does not satisfy $M^*$.

The simulator then implicitly defines $t$ by randomly choosing a number $r \in \mathbb{Z}_p$ as

$$t_{uid,k} = r + w_1 q r^{-1} + w_2 a r^{-2} + \cdots + w_{mp} a^{r-mp}$$

by setting

$$L_{uid,k,x_k} = (g^{u'_{uid}})^{t_{uid,k}} = \prod_{i=1}^{n} \frac{g^{u'_{uid}}}{g^{r_{uid}}} = g^{8_{uid} x_k}.$$

The simulator then constructs $R_{uid,k}$ as

$$R_{uid,k} = g^{ar} \cdot \prod_{i=1}^{n} (g^{a r^{i+1-i}})^{w_i}.$$

From the definition of $g^{u'_{uid}}$, we find that $g^{u'_{uid}}$ contains a term of $g^{a r^{i+1-i}}/a_{uid}$, which will cancel out with the unknown term in $g^{8_{uid} x_k}$ when creating $K_{uid,k}$. The simulator can calculate

$$K_{uid,k} = g^{ar} g^{u'_{uid}} g^{8_{uid} x_k} \cdot \prod_{i=1}^{n} (g^{a r^{i+1-i}})^{w_i}.$$
we can let
\[ K_{\text{uid},x_k} = (L_{\text{uid},k})^{\gamma_k} \cdot (\text{GPK}_{\text{uid}})^{\delta_k} \gamma_k. \]

Towards update key queries, suppose the adversary submits pairs of \( \{ (\text{uid},x_k) \} \). If the attribute \( x_k \) has a new version number \( v_k' \), and \( \text{uid} \) is a non-revoked user, it then sends back the key update key as
\[ K_{U\text{K},x_k} = (g^{v_k} \cdot \beta_k)^{v_k'} = (g^{v_k - v_k} \cdot \beta_k^{v_k'}) \cdot (g_{v_k'})^{(v_k - v_k')} \).

Otherwise, it responses “L”.

**Challenge.** In this phase, the simulator programs the challenge ciphertext. The adversary gives two messages \( m_0, m_1 \) to the simulator. The simulator flips a coin \( b \). It creates
\[ C = m_b T \cdot \prod_{k \in \mathcal{I}_k} e(g^s, g^{\text{auid}_k}) \]
and \( C' = g^s \).\( C'' = g^{\delta_k} \).

The difficult part is to simulate the \( C_i \) values since this contains terms that must be canceled out. However, the simulator can choose the secret splitting, such that these can be canceled out. Intuitively, the simulator will choose random \( y_2', \ldots, y_n' \), and share the secret \( s \) using the vector
\[ v = (s, sa + y_2', sa^2 + y_3', \ldots, sa^{n-1} + y_n') \in \mathbb{Z}_p^n. \]
It also chooses random values \( r_1', \ldots, r_i' \).

For \( i = 1, \ldots, n' \), let \( R_i \) be the set of all \( k \neq i \) such that \( \rho^*(i) = \rho^*(k) \). That is the set of all other row indices that have the same attribute as row \( i \). The challenge ciphertext components can be generated as
\[ D_{1,i} = \left( g^{r_i g^{v_k} \delta_k} \right)^{\frac{1}{R_i}} \quad D_{2,i} = \left( g^{r_i' g^{v_k} \delta_k} \right)^{\frac{1}{R_i}}. \]

From the vector \( v \), we can construct the share of the secret as
\[ \lambda_i = s \cdot M_{1,'} + \sum_{j=2,\ldots,n'} (sa_i^{-1} + y_j') M_{j,'}. \]

Then, we can simulate the \( C_i \) as
\[ C_i = (g^{r_i g^{v_k} \delta_k} \cdot H(\rho^*(i))) \cdot \left( \prod_{j=1,\ldots,n'} e(g^{M_{j,'}}, g^{\lambda_j}) \right). \]
\[ = \left( g^{r_i g^{v_k} \delta_k} \cdot \prod_{j \in R_i} e(g^{M_{j,'}}, g^{\lambda_j}) \right). \]

**Phase 2.** Same as Phase 1.

**Guess.** The adversary will eventually output a guess \( b' \) of \( b \). If \( b' = b \), the simulator then outputs 0 to show that \( T = e(g, g)^{\text{ad}^{(i+1)}} \); otherwise, it outputs 1 to indicate that it believes \( T \) is a random group element in \( \mathbb{G}_T \).

When \( T \) is a tuple, the simulator \( B \) gives a perfect simulation so we have that \( Pr[B(\vec{y}, T, e(g, g)^{\text{ad}^{(i+1)}}) = 0] = \frac{1}{2} + \text{Adv}_{B_1} \).

When \( T \) is a random group element the message \( m_b \) is completely hidden from the adversary and we have at \( Pr[B(\vec{y}, T, e(g, g)^{\text{ad}^{(i+1)}}) = 0] = \frac{1}{2} \).

Therefore, \( B \) can play the decisional q-parallel BDHE game with non-negligible advantage.