The object of this experiment is to measure the moment of inertia of a ring and a disc and compare the experimental values to the theoretical values calculated from the mass and dimensions of each body.

Theory

Imagine three identical bricks of edge lengths a, b and c which are rotated about an axis parallel to the respective edges and passing through the center of mass. Each brick is rotating with the same angular velocity $\omega$.

The amount of work required to bring the bricks to rest is different for all three cases. Evidently, the total kinetic energy of each block, that is, the work that must be done to bring the bricks to rest, depends on more than the mass of the brick and its angular velocity $\omega$.

In other words, the tendency of an object that is rotating about some axis to keep rotating depends on the mass of the object and the way the mass is distributed about the axis of rotation. This tendency is called rotational inertia or moment of inertia, I.

In order to gain a better understanding of this concept, consider a rotating cylinder as shown in figure 2. The angular velocity of $m_1$ and $m_2$ is the same $\omega$ but the linear velocities of the masses are different.

The kinetic energy of each mass is given by:

$$KE_1 = \frac{1}{2} m_1 v_1^2 \quad \text{and} \quad KE_2 = \frac{1}{2} m_2 v_2^2$$

The velocities are:

$$v_1 = \omega r_1 \quad \text{and} \quad v_2 = \omega r_2$$

So the kinetic energy can be rewritten as:

$$KE_1 = \frac{1}{2} m_1 (\omega r_1)^2 \quad \text{and} \quad KE_2 = \frac{1}{2} m_2 (\omega r_2)^2$$

The total energy of the rotating cylinder can be found by summing over all the mass elements:
The kinetic energy of a body in translational motion is \( KE = \frac{1}{2} \sum_i m_i v_i^2 = \frac{1}{2} \sum_i m_i (\omega r_i)^2 \)

\[ KE = \frac{1}{2} \left\{ \sum_i m_i r_i^2 \right\} \omega^2 \]

The kinetic energy of a body in translational motion is \( \frac{1}{2}mv^2 \), where \( m \) is the resistance to a change in velocity. By analogy then, we define the sum inside the brackets to be the rotational inertia or moment of inertia of that body about that particular axis of rotation.

\[ I = \sum_i m_i r_i^2 \]

The moment of inertia summation can be evaluated for a few simple cases without too much difficulty. For example, a single mass \( m \) traveling in a circle of radius \( r \) has a moment of inertia of \( mr^2 \). Two masses \( m \) revolving about their center of mass and a distance \( r \) from the center of mass have a total moment of inertia of \( 2mr^2 \) or \( m'r^2 \) where \( m' \) is the total mass. A further extension summation yields \( mr^2 \) as the moment of inertia of a thin hoop of mass \( m \) and radius \( r \) revolving about an axis passing through the center of mass and perpendicular to the plane of the hoop.

In most cases, integral calculus is used to compute the rotational inertia of more complex shapes. For example, the moment of inertia of a disc is \( I_D = \frac{mR^2}{2} \) and the moment of inertia for a ring is \( I_R = \frac{m}{2(R_1^2 + R_2^2)} \) where \( R_1 \) is the inner radius and \( R_2 \) is the outer radius. Both of these relations are for an axis passing through the center of mass and parallel to the cylinder axis.

It should be noted that since the moment of inertia is additive, two bodies bound together have a total moment of inertia about an axis equal to the sum of their individual moments about that axis \( (I = I_1 + I_2) \).
In this experiment a turntable will be caused to rotate by a falling mass $m$. A ring or a disc will be placed on the turntable and the moment of inertia will be determined experimentally for both. A diagram of the apparatus and some of the quantities to be measured is shown in figure 3. Underneath the turntable, and attached to it, is a drum of radius $r$. A cord is wrapped around the drum and runs over the two pulleys to the mass $m$. The mass falls a distance $h$ during time $t$. Since the acceleration, $a$, is constant, we may write:

$$h = \frac{1}{2} at^2$$

or

$$a = \frac{2h}{t^2}$$

also

$$v = at = \frac{2h}{t}$$

From the conservation of energy we know that:

$$\Delta PE = KE_{falling\ mass} + KE_{rotation}$$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

Since $\omega = \frac{v}{r}$ we may rewrite the last equation as:

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\left(\frac{v}{r}\right)^2$$

Solving for $I$ and using $v = \frac{2h}{t}$:

$$I = mr^2 \left\{ \frac{gt^2}{2h} - 1 \right\}$$

This result is valid if there is no friction; however, this is not the case and we must consider frictional effects.
The effect of friction may be accounted for in the following manner. Arrange the apparatus so the mass $m$ will fall and then rebound as the rotational inertia causes it to continue to rotate in the same direction. If the mass falls from an original height $h_0$ above the floor and then rebounds to a height $h_r$, the difference in the potential energy of $m$ will be $mg(h_0 - h_r)$. This is the work done by the frictional forces. The mass will have traveled a total distance of

$$(h_o - h_1) + (h_r - h_1) = h_0 + h_r - 2h_1$$

where $h_1$ is the distance between the floor and the lowest position of $m$.

The work per unit distance is:

$$w_f = \frac{mg(h_0 - h_r)}{h_0 + h_r - 2h_1}$$

The conservation of energy equation now has an additional term to account for frictional losses as the mass falls a distance $h_o - h_1$.

$$mg(h_0 - h_1) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + (h_0 - h_1)\left(\frac{mg(h_0 - h_r)}{h_0 + h_r - 2h_1}\right)$$

Solving for $I$ and simplifying gives:

$$I = mr^2\left\{\frac{gt^2}{2(h_o - h_1)}\left[1 - \frac{h_0 - h_1}{h_o - h_r - 2h_1}\right] - 1\right\}$$

where $I$ is the moment of inertia of the turntable alone or the turntable plus whatever load is placed on it.

**Procedure**

1. Measure the diameter of the drum with the vernier caliper. Make certain the cord is tied tightly to the drum and does not slip.

2. Position the turntable so that the falling mass does not strike the floor. Attach a 50-gram weight hanger to the cord and wind the cord on the drum (do not allow the cord windings to overlap) until the weight hanger is about 200 cm above the floor. Record this height as $h_o$. Release the weight and time the fall from $h_o$ to $h_1$. Allow the weight to rebound and place your finger on the turntable the instant it stops rotating. Measure the rebound height $h_r$. Finally, record the distance from the floor to the weight hanger at its lowest point $h_1$. Repeat these steps three times and average the results.

3. Use the equation for $I$ to calculate the moment of inertia of the turntable.
4. Repeat steps 2 and 3 with the disc on the turntable and calculate the moment of inertia of the disc.

5. Repeat steps 2 and 3 with the ring on the turntable and calculate the moment of inertia of the ring.

6. Calculate the theoretical values for the moments of inertia for the ring and the disc. Calculate the percent error for each assuming the theoretical value to be correct.