Experiment 9

Centripetal Force

The object of this experiment is to understand the motion of a particle traveling in a circle at constant speed. In particular, we wish to define and verify the expressions for centripetal acceleration and centripetal force.

Theory

Newton and Galileo both observed that a body in motion would travel in a straight line unless acted upon by an external force. Consider, for example, a ball rolling across the floor in a straight line. Suppose that we lay some boards on the floor in such a way that the ball will be deflected into a closed path (Figure 1). The boards will be arranged in such a way as to always exert a force on the ball somewhat toward the center of the closed path. In fact, if a circle replaced the boards, the force would be directed toward the center of the circle. This force is called the centripetal force.

The centripetal force is not directed along the circular path but perpendicular to the path. Thus, the centripetal force does not do any work on the ball and it can cause no change in the kinetic energy. The acceleration due to the centripetal force causes a change in the direction of the velocity vector but no change in the magnitude of the velocity vector.

Consider a body of mass $m$ moving in a circular path or radius $r$ (Figure 2). At one instant the velocity at point A is $\vec{v}_1$. At a later time, $\Delta t$, the velocity at point B is $\vec{v}_2$. It is left as an exercise for the student to show that the triangles OAB and $v_1$, $v_2$, $\Delta v$ are similar.

Figure 1

Figure 2
From the OAB triangle we see that for small $\Delta \theta$

$$\Delta s \approx r \Delta \theta \quad (1)$$

and

$$v = \text{speed} = \frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t}$$

or

$$\Delta \theta = \frac{v}{r} \Delta t \quad (2)$$

From the velocity triangle, we see that

$$\Delta v = v \Delta \theta$$

or

$$\Delta \theta = \frac{\Delta v}{v} \quad (3)$$

Substituting equation (3) into equation (2) yields

$$\Delta \theta = \frac{\Delta v}{v} = \frac{v}{r} \Delta t \quad (4)$$

Re-arranging equation (4) and using the definition of acceleration produces

$$a_c = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$a_c = \frac{v^2}{r}$$

This is the centripetal acceleration, which acts on the object in circular motion at constant speed, and is directed toward the center of the circle. The centripetal acceleration times the mass of the object moving in a circle is the centripetal force.

$$F_c = ma_c = \frac{mv^2}{r} \quad \text{centripetal force}$$

It should be understood that the centripetal force is not a force in itself (like gravity or electricity) but rather it is the amount of force that must be exerted to keep the mass $m$ with speed $v$ moving in a circle of radius $r$. 

Another way of expressing the centripetal acceleration may be found by defining the rate at which angle is being swept out as \( m \) moves about the circle.

\[
\omega = \textit{angular velocity} = \frac{\Delta \theta}{\Delta t}
\]

Then, from equation (2)

\[
\omega = \frac{v}{r} \quad \text{or} \quad v = \omega r
\]

We may substitute for \( v \) in the expression for centripetal acceleration.

\[
a_c = \frac{v^2}{r} = \left(\frac{\omega r}{r}\right) = \frac{(\omega r)^2}{r}
\]

\( \theta \) is measured in radians and thus \( \omega \) is in radians per second.

Another useful relation is found by introducing the frequency \( f \). Let \( T \) equal the time per revolution. Then

\[
v = \frac{2\pi r}{T} = 2\pi f r
\]

where \( f \) = frequency = number of revolutions per unit time.

\[
\omega = \frac{v}{r} = 2\pi r
\]

and

\[
a_c = \omega^2 r = 4\pi^2 f^2 r
\]

\[
F_c = ma_c = 4m\pi^2 f^2 r
\]
Example: A beetle of mass 10 grams is standing on a record turntable revolving with a frequency of 45 rpm. If the beetle is 10 cm from the axis of rotation what is the minimum force keeping the beetle from slipping?

\[
f = \frac{45 \text{ rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{0.75 \text{ rev}}{\text{sec}}
\]

\[
F = 4m\pi^2f^2r = 4 \times 10 g \times \pi^2 \times (0.75)^2 \times \frac{10\text{cm}}{\text{sec}^2} = 2221 \text{ dynes}
\]

You may be wondering about the disappearance of the unit revolutions. One complete revolution equals an angle of \(2\pi\) radians. Thus, the quantity \(2\pi f\) has dimensions of \(2\pi\) radians/revolution times \(f\) revolutions/time or \(2\pi f\) radians/time. The radian is really dimensionless and is only included to indicate that the angle is being measured correctly.

**Procedure**

This experiment will consist of two parts. First, we will keep the centripetal (literally the center-seeking force) force on the rotating mass, \(M\), constant and vary the radius. In the second part, we will hold the radius constant and change the amount of force required to keep the object moving in a circular path.

The equipment consists of a rotating platform, a center post assembly with a spring and orange indicator button and an indicator position bracket, a side post assembly which supports the mass which will move in the circle and a clamp-on pulley at the end of the rotating platform that will support the hanging weights. See the figure below.

![Figure 1](image-url)
**Constant Force**

1. Weigh the object and record its mass, $M$, on the data sheet. Reattach the object to the side post bar and set the side post at the 16 cm mark on the platform scale. Move the sprint support on the center post assembly to the top of the slot in the center post. Attach the clamp-on pulley to the end of the rotating platform nearest the side post assembly and tie a 5.0-gram mass hanger to the string that runs over the clamp-on pulley to the object. Make sure the string from the object to the pulley is horizontal by adjusting the length of the string supporting the object. Add enough mass to the hanger to bring the total hanging mass up to 60 grams and record this as the hanging mass, $m$, on the data sheet. This mass multiplied by the acceleration of gravity establishes the constant centripetal force.

2. Next, adjust the side post position on the rotating platform until the threads supporting the rotating mass are perfectly aligned with the vertical line on the side post. Record the radius of the circle as read off the scale on the rotating platform in Table 1 on the data sheet. Finally, align the indicator bracket on the center post with the orange indicator button. Note: The apparatus you are using is highly variable and many of the settings for the radius and/or the weights and/or the position of the spring holder bracket are suggestions and not commands. Please ask your lab instructor if you cannot align the object supports.

3. Remove the hanging mass and rotate the platform by spinning the knurled post under the platform. Slowly increase the rate of rotation until the orange indicator button is centered in the indicator bracket. This means that the strings supporting the rotating object are once again vertical and the object is rotating in a circle at the predetermined radius. Maintain this rate of rotation and measure the time required for ten complete rotations. Now take the ten revolutions and divide it by the time you measured. This is the frequency, $f$, which is the number of revolutions per second. Record the frequency on the Data Sheet in Table 1. Also square the frequency and record this value in the next box in the data table. Note: The measurement of the frequency is critical to obtaining satisfactory results.

4. Use your experiment data to complete Table 1. Repeat the steps 1 through 3 two more times for different radii. Use any radii you wish but they must be at least 1 cm different than any other radius used.

**Constant Radius**

5. In this part of the experiment we will hold the radius constant and vary the centripetal force. Move the sprint bracket to the top position in its slot. Set the side assembly at a radius of 18 cm. Now attach the mass hanger and add enough mass (use as many 20 gram masses as possible) to align the support strings for the object with the vertical line on the side post. Note: If the strings will not align with the masses furnished, allow the strings to lean slightly to the right and align the strings by moving the spring bracket down slightly. Record the total hanging mass in Table 2.

6. Align the indicator bracket on the center post with the orange indicator button. Remove the hanging mass and begin the rotation and once the orange indicator button is in the indicator bracket, time ten revolutions and record the frequency in Table 2 (Remember, the frequency is the 10 revolutions divided by the time on the stopwatch, as noted in step 3).
7. To vary the centripetal force, reattach the mass hanger and weights to the string that runs over the pulley. Remove one of the masses from the hanger (try a 20 gram mass) and record the new hanging mass in Table 2. Adjust the spring bracket downward until the suspension strings are vertical are again vertical. Repeat the procedure in step 6.

8. Repeat steps 6 and 7 with a different hanging mass one more time.

9. Finish the exercises on the computer.