Comp 2700 (Discrete Structures) Fall 2018. Homework 1.

Submissions: This assignment is due in class on Wednesday Sep 12th, 2018. Each student must submit his or her own assignment. Solutions can either be typed in Latex, MSWord or other such word processing software, or printed clearly. You must write your name and UUID clearly on your submitted assignment.

Academic Integrity: You are encouraged to work in groups, but everyone must write out their own solutions. Absolutely no word to word copying is allowed. Please refer to the course policies and schedules about this. If you have worked with other students on the assignment or referred to external sources, please mention all names and sources on your assignment.

Partial solutions: If you are sure that you know how to arrive at a solution, but you get stuck in some place, it is better to write the partial solution. Honest attempts at partial solutions will be awarded.

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Problem 1 [5 + 5 pts]: Construct the truth table for the following compound propositions:

(a) \((p \leftrightarrow q) \lor (\neg p \leftrightarrow q)\)
(b) \(((p \rightarrow q) \rightarrow r) \rightarrow s\)

Problem 2 [5 + 5 pts]: Show that the following are tautologies by using truth tables:

(a) \([((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)\]
(b) \([([p \lor q] \land (p \rightarrow r) \land (q \rightarrow r)] \rightarrow r\]

Problem 3 [10 + 10 pts]: The same problem, but now derive them using rules for equivalence.

Problem 4 [20 pts]: We define a logical operator called the Pierce arrow denoted \(\downarrow\) as follows: \(p \downarrow q\) is True when both \(p\) and \(q\) are False, and it is False otherwise. Find a compound proposition equivalent to \(p \rightarrow q\) using only the logical operator \(\downarrow\).

Problem 5 [5 + 5 pts]: For each part below, all the quantifiers have the same nonempty domain. Show that

(a) \(\forall x P(x) \land \exists x Q(x)\) is logically equivalent to \(\forall x \exists y (P(x) \land Q(y))\)
(b) \(\forall x P(x) \lor \exists x Q(x)\) is logically equivalent to \(\forall x \exists y (P(x) \lor Q(y))\)

Problem 6 [5 + 5 pts]: If it is hot today and it rained yesterday, then I will be miserable today. If it is not hot today, then I am going to play soccer. If it did not rain yesterday, it will not rain today. I am not miserable today. Therefore, it did not rain today or I am going to play soccer.

(a) Model the above given conditions as logical statements. Clearly define the propositions. Also, indicate the conclusion.
(b) If the reasoning is correct, prove it using the rules of inference for propositional logic.

Problem 7 [20 pts]: Let the domain of discourse \(D\) be a group of people that includes men and women. Let \(M(x)\) be the predicate “\(x\) is a man”, and let \(W(x)\) be the predicate “\(x\) is a woman”. Let \(T(x, y)\) be a predicate “\(x\) is married to \(y\)”. Formulate a statement in first order logic that is true if and only if each man in the group is married to exactly one woman in the group, and each woman in the group is married to exactly one man in the group.