
Submissions: This assignment is due in class on Monday Feb 18th, 2019. Each student must submit his or her own assignment. Solutions can either be typed in Latex, MSWord or other such word processing software, or printed clearly. You must write your name and UUID clearly on your submitted assignment.

Academic Integrity: You are encouraged to work in groups, but everyone must write out their own solutions. Absolutely no word to word copying is allowed. Please refer to the course policies and schedules about this. If you have worked with other students on the assignment or referred to external sources, please mention all names and sources on your assignment.

Partial solutions: If you are sure that you know how to arrive at a solution, but you get stuck in some place, it is better to write the partial solution. Honest attempts at partial solutions will be awarded.

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Problem 1 [20 pts]: Problem 50 from the Exercises in Sec 2.2. The problem is reproduced below. 

Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer $i$,

(a) $A_i = \{i, i+1, i+2, \ldots\}$.
(b) $A_i = \{0, i\}$.
(c) $A_i = (0, i)$, that is, the set of real numbers $x$ with $0 < x < i$.
(d) $A_i = (i, \infty)$, that is, the set of real numbers $x$ with $x > i$.

Problem 2 [30 pts]: Problem 34 from the Exercises in Sec 2.3. The problem is reproduced below.

If $f$ and $f \circ g$ are one-to-one, does it follow that $g$ is one-to-one? Justify your answer.

Problem 3 [20 pts]: Problem 74 from the Exercises in Sec 2.3. The problem is reproduced below.

Prove or disprove each of these statements about the floor and ceiling functions.

(a) $\lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$ for all real numbers $x$.
(b) $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ for all real numbers $x$ and $y$.
(c) $\lceil \lfloor x/2 \rfloor / 2 \rceil = \lfloor x/4 \rfloor$ for all real numbers $x$.
(d) $\lceil \sqrt{\lfloor x \rfloor} \rceil = \lfloor \sqrt{x} \rfloor$ for all positive real numbers $x$.
(e) $\lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor \leq \lfloor 2x \rfloor + \lfloor 2y \rfloor$ for all real numbers $x$ and $y$.

Problem 4 [10 pts]: Problem 36 from the Exercises in Sec 2.4. Problem is reproduced below.

Use the identity $1/(k(k+1)) = 1/k - 1/(k+1)$ and Exercise 35 to compute $\sum_{k=1}^{n} 1/(k(k+1))$.

Problem 5 [20 pts]: Problem 14 from the Exercises in Sec 2.6. Problem is reproduced below.

The $n \times n$ matrix $A = [a_{ij}]$ is called a **diagonal matrix** if $a_{ij} = 0$ when $i \neq j$. Show that the product of two $n \times n$ diagonal matrices is again a diagonal matrix. Give a simple rule for determining this product.